# INFLUENCE OF DISCRETE INDUCTANCE CURVES ON THE SIMULATION OF A ROUND ROTOR GENERATOR USING COUPLED CIRCUIT METHOD

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**Abstract** – This paper presents a study on the influence of the discretisation of the inductance curves on a detailed coupled circuit model of a synchronous generator with a damper winding and search coils. The self and mutual inductances of all coupled circuit are computed in magnetostatic with a 2D finite-element method (FEM) for different rotor positions. The circuit model is solved in Matlab-Simulink. Two inductance models have been developed and comparisons are made regarding the number of discrete rotor positions used to evaluate the inductance curves by the FEM. The first one is based on a Fourier transformation on the discrete inductance curves in order to obtain continuous curves. The other method uses linear interpolation in the inductances and derivative of inductances tables. The simulation and experimental results. It is shown that the Fourier transformation method is too time-consuming; a trade-off on the sampling of inductance curves must be done in regard to the harmonic content of the results and simulation time.

**Keywords** – Coupled circuit, Round rotor generator, Damper bars, Inductance identification, Harmonic content

### **1. INTRODUCTION**

In this paper, we are interested in the problem of evaluating the damper bars currents of synchronous machines. Such information is important for machine designers when high level of confidence has to be established on the machine performances or for a diagnostic. The problem associated with the evaluation of the damper bars currents is the time required to solve the simulation with modern finite element simulation software, especially when we are interested in steady state current waveforms which require magnetodynamic simulations.

To reduce the simulation time and rapidly reach the steady state, it is possible to use analytical models that can represent the damper winding currents. Several works have been done on models based on permeance functions and models using equivalent circuit with different levels of simplification as shown in [1].

The use of permeance functions is a simplified approach to finite-elements that may represent the structure of the machine [2]–[4]. The drawback of permeance functions is that it might be complicated to evaluate an accurate function that represents the high order harmonics when there are sharp permeance variations or complex shapes. When there is a need for an accurate representation of the machine's geometry, finite-elements methods should be used to evaluate the flux linkage between windings in magnetostatic simulations. The coupled circuit method is a simplified approach that can be applied to machines studies. Instead of representing the links through permeance functions, the coupling between the windings is expressed with inductances. Again, the linkage can be represented with winding functions evaluated in manners similar to the permeance functions [5] or with finite-elements [6].

These methods applied to synchronous machines are mainly aimed at predicting the no-load voltage waveform [4], [6]. To accurately predict the noload voltage with a high harmonic content, the exact magnetic shape of the generator must be considered as well as the currents in the damper bars. The stator winding distribution must also be included in order to consider all space harmonics. This proved necessary to verify that large salient pole generators complies with the Telephone Influence Factor (TIF) requirements [7]. This forbids the use of simplified models such as those based on rotor transformations (dq models) [8], but they might still remain useful for diagnostics on the rotor.

In this work, we used the detailed coupled circuit method and the inductances were evaluated by finite-element simulations for different rotor positions [6]. The analytical model was built in Matlab-Simulink and we used Flux2D-Cedrat for the finite-element simulations. We show a study on the effects of the angular discretization of the inductances in the finite-element software. We are interested in finding how the level of discretization affects the shapes of the different inductance curves and their harmonic content. We then use these inductance curves in our detailed coupled circuit using two approaches.

The first approach consists of using a Fourier transformation in order to obtain continuous equations that define the inductances from the tabulated values. The Fourier transformation approach allows an exact solution to the inductance derivative that may be used in the time-stepping simulation. This eases the use of classical integration techniques available in Matlab-Simulink.

The other approach is to use directly the inductance and pre-calculated inductance derivative tables in the Simulink simulation. To use the classical integration techniques with a variable time-step, we implemented linear interpolation inside the tables.

The results of the two approaches are compared on the basis of simulation time and the accuracy of the waveforms. These results are then compared to the experimental results.

## 2. GENERATOR USED FOR THE STUDY

The machine used for this study is a three phases round rotor synchronous generator of 5.4 kVA. The stator has 54 slots. The rotor has 3 concentric coils per pole and a total of 24 copper damper bars connected by short-circuit rings.

The stator lamination stack is held in place by thru bolts in the tooth tip. By removing thru bolts, we can insert a wire and form a search coil. Two search coils having different slot pitch are shown on the Fig. 1. The winding of this machine is of a special construction; it is made of rectangular conductors and there are 14 conductors per slot distributed in four layers. They are soldered in the extremity in order to put the individual conductors in series to form coils. The two bottom layers have four individual conductors and top layers (near the air gap), three individual conductors.



Fig. 1: Stator with the search coils

It can be seen that the stator slots are almost closed near the air gap; there is no retaining insulation near the air gap, it's the tooth tip that retains the windings.

This machine was designed to allow easy modifications to the electrical transient behaviour. As it can be seen on the Fig. 2, the dampers can be unbolted from each short-circuit ring. They are not soldered in place, which would have been the most cost effective way.



Fig. 2: Rotor with the damper bars

The 2D sketch of half the machine is shown on Fig. 3. It is the finite-element domain used since there is a periodicity on half the machine domain. It can be seen that the two search coils are also modelled in the tooth tip.



Fig. 3: Sketch of half the machine

### 3. COUPLED CIRCUIT MODEL

We chose to represent the behavior of this machine with 30 coupled circuits: 3 circuits for the stator phases, 1 circuit for the field, 24 circuits for the damper bars and 2 circuits for the search coils. Each linear circuit is modeled by a resistance and several inductances. The inductances depend on the angular position  $\theta$  of the rotor.

$$[V] = [R][I] + \frac{d\{[L(\theta)][I]\}}{dt}$$
(1)

To represent the movement and the variation of current, we may express (1) in the following manner where  $\Omega$  is the rotor angular speed.

$$[V] = [R][I] + \Omega \left\{ \frac{d[L(\theta)]}{d\theta_m} \right\} [I] + [L(\theta)] \left\{ \frac{d[I]}{dt} \right\}$$
(2)

The following expression provides the derivative of the current. The voltage on each circuit must be imposed to solve this equation. We assumed that the voltage of each damper bar is null. However, these circuits can easily be connected to each other by impedances to simulate an end-ring circuit [6].

$$\frac{d[I]}{dt} = [L(\theta)]^{-1} \cdot \left[ [V] - \left( [R] + \Omega \frac{d[L(\theta)]}{d\theta_m} \right) [I] \right]$$
(3)

#### 4. INDUCTANCE CURVES IDENTIFICATION

The first step of this modelling method is to identify the inductance curves for a given number of rotor positions. To evaluate these inductances, we preferred to use a magnetic field computation based on finite elements for the aforementioned reasons. The magnetic saturation of the material was neglected. Fig 4 illustrates a magnetostatic FE simulation and the corresponding flux densities in the machine with the nominal current in the field winding.



Fig. 4: Flux densities

The number of rotor positions should be chosen high enough to allow the representation of the noload voltage harmonic content. As explained in [9], 20 rotor positions within one stator slot pitch allows generally a very precise prediction of the harmonics due to the slot pulsation field. We evaluated the impact of such a rule on the round rotor generator considered in this study. The inductances have been identified for 1080 rotor angular positions per turn (20 rotor positions per stator slot pitch). To analyze the influence of the discretization on the inductances curves, we also performed FE simulations using 540 positions per turn (10 rotor positions per stator slot pitch) and 270 positions per turn (5 rotor positions per stator slot pitch).

We studied a Fourier transformation approach since it allows a continuous solution to the inductance and inductance derivative curves that may be used in the time-stepping simulation. This eases the use of classical integration techniques with a variable time-step. The relevant frequency content of the Fourier expansion was evaluated from a harmonic analysis of the curves. Then, curve fitting techniques were used to evaluate the parameters  $(a_0, a_i, b_i)$  of the analytical functions.

$$L(\theta) = a_0 + \sum_i a_i \cdot \cos(i \cdot \theta) + b_i \cdot \sin(i \cdot \theta)$$
(4)

$$\frac{dL(\theta)}{d\theta} = \sum_{i} -a_{i} \cdot i \cdot \sin(i \cdot \theta) + b_{i} \cdot i \cdot \cos(i \cdot \theta)$$
(5)

Fig. 5 shows the field winding self-inductance. It was expected to be a flat line but it revealed to be the signal that highlights the highest frequency (108 oscillations per turn). That was really useful information that we used in this study. With the inductance curve made of 1080 rotor positions, there is some aliasing, but it is more easily seen on the curve with 270 rotor positions. In the Fourier expansion, the harmonic orders 108, 216 and 432 were used.



Fig. 5: Discrete and analytical curves of the field self-inductance

Fig. 6 shows the derivative of the field winding self-inductance. The same harmonic orders were used. Again, the sampling at 270 rotor positions removes a lot of information on the harmonic content.



Fig. 6: Discrete and analytical curves of the field self-inductance derivative

Fig. 7 shows the mutual inductance curve between the field winding and a stator phase. There is no DC component and the fundamental frequency is the same as the electrical frequency. The harmonic content appears to be small, but the tooth ripple frequency is easily seen on the derivative of that inductance on Fig. 8. The Fig. 9 gives a zoom of the Fig. 8. That derivative is linked to the no-load voltage waveform, so the harmonic content has to be considered in the Fourier expansion. We chose to use these harmonic orders: 2, 4, 6, 8, 54, 108 and 216.



Fig. 7: Discrete and analytical curves of the field to phase mutual inductance



Fig. 8: Discrete and analytical curves of the field to phase mutual inductance derivative



Fig. 9: Zoom on the discrete and analytical curves of the field to phase mutual inductance derivative

### 5. TIME-STEPPING SIMULATION METHOD

To be able to use the discrete inductance curves in a variable time-stepping simulation, we had to provide a mean to evaluate the values at any angular position. As explained before, we compared two methods: the Fourier transformation and the linear interpolation methods.

The Fourier transformation appeared to be an elegant solution to determine continuous functions for the inductance and inductance derivative curves. It removes some aliasing effects that may arise when too little angular positions are used in the identification of the curves. This can be done since the harmonic orders can be chosen. The drawback of using the analytical expressions in the time-stepping simulation is that the simulation time revealed to much higher than the interpolation method. To benefit from the anti-aliasing effect without having the burden of the analytical expressions in the time-stepping simulation is that the simulation is possible to re-sample the Fourier curves, but this was not performed in this study.

The interpolation method was implemented using a linear interpolation technique. We chose this technique for the sake of simplicity, but we could have used a spline interpolation as in [10]. That interpolation was done on the inductance table and on the derivative and second derivative inductance tables. The following expression shows the linear interpolation technique:

$$L(\theta) = \frac{L(\theta_i) - L(\theta_{i-1})}{\theta_i - \theta_{i-1}} \cdot \theta + L(\theta_{i-1})$$
(6)

In Matlab-Simulink, it is possible to use a wide variety of integration techniques. We used variablestep and fixed-step techniques for unloaded and loaded operation. To keep the simulation stable, we had to respect a maximum time-step in the simulation. The maximum time-step that reveled to give stability had to be 10 times smaller than the equivalent time-step given by the angular discretization. For the Fourier transformation method, the given maximum time-step was the same ( $\Delta t_{max}$ ), but we can express that same rule with the maximum harmonic order (*h*) instead of the number of points per turn (*n*). The angular speed in radian per second is ( $\alpha$ ).

$$10 \cdot \Delta t_{max} < \frac{2\pi}{\Omega \cdot n} \tag{7}$$

$$10 \cdot \Delta t_{max} < \frac{2\pi}{\Omega \cdot 2h} \tag{8}$$

The simulation time results for one electrical period of no-load operation at 1800 rpm are shown in table I. The Fourier transformation method takes roughly 30 times more time than the simulation with interpolation within the table of 1080 points per turn. It can be seen that increasing the discretization interval reduces the simulation time by allowing a larger time-step. Consequently, there is some loss of precision in the simulation results.

Inductance model	Simulation time [s]
Fourier series	1735
1080 pts curves	52
540 pts curves	38
270 pts curves	26

I. Simulation t	ime compa	rison
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The Fig. 10 shows the no-load line to neutral EMF waveform obtained with Matlab-Simulink. It can be seen that the overall shape is well preserved even when using 270 points per turn. The loss of precision is on the high harmonic content, as can be seen on the Fig. 11. It has to be stated that the curve with the 1080 points gives the same result as the finite-element transient simulation performed without magnetic saturation.







Fig. 11: Zoom on computed no-load voltage

#### 6. EXPERIMENTAL VALIDATIONS

The Fig. 12 and Fig. 13 shows the induced voltage in the search coil of 2 and 10 slots pitch respectively. It can be seen that the characteristic shape is quite well preserved. There is an issue with the amplitude of the harmonics but it is the same problem with the results of the finite-element simulation software..









The Fig. 14 shows the steady state current in one stator phase with each phase of the stator in shortcircuit with itself. It can be seen that there is the presence of a third harmonic. That third harmonic would not be there if it was a line-to-line three phases short-circuit. To obtain that curve, we had to simulate 4 seconds in order to obtain the steady state. That simulation took a few hours to compute. It would have been unthinkable to do the same simulation with the finite-element software. The Fig. 15 shows some damper currents during that same simulation on the same time frame.



Fig. 14: Steady-state phase current during the short-circuit test



Fig. 15: Calculated damper currents during the short-circuit test

#### 7. CONCLUSION

This work has shown that the Finite Element method is interesting to evaluate the inductance curves with a desired level of precision and that the higher harmonic content can be readily seen on the curve of the field self-inductance For analysis purposes, the time required for this identification is quite well compensated. It was shown that the Fourier expansion method is interesting to remove unwanted harmonic content but is too much time consuming when used with the analytical equations in the time-stepping resolution. The results have been confirmed by the experience: waveforms have been repeated with some errors on the amplitudes.

### REFERENCES

 R. Casimir, E. Bouteleux and al., « Comparison of modelling methods and of diagnostic of asynchronous motor in case of defects », in *Power Electronics Congress*, 2004. CIEP 2004. 9th IEEE International, 2004, p. 101-108.

- [2] J. J. Rocha, "The influence of damper winding slot pitch upon losses and synchronising torque », Int. Conf. Evol. Mod. Asp. Synchronous Mach., vol. Part I, p. 636-641, août 1991.
- [3] A. M. Knight, H. Karmaker, and K. Weeber, « Use of a permeance model to predict force harmonic components and damper winding effects in salient pole synchronous machines », in *Electric Machines and Drives Conference, 2001. IEMDC 2001. IEEE International*, 2001, p. 179-184.
- [4] G. Traxler-Samek and A. Schwery, « Analytic calculation of the voltage shape of salient pole synchronous generators including damper winding and saturation effects », *Int. J. Comput. Math. Electr. Electron. Eng.*, vol. 22, nº 4, p. 1126-1141, 2003.
- [5] G. Houdouin, G. Barakat, B. Dakyo, and E. Destobbeleer, « A winding function theory based global method for the simulation of faulty induction machines », in *Electric Machines and Drives Conference, 2003. IEMDC'03. IEEE International*, 2003, vol. 1, p. 297-303 vol.1.
- [6] S. Keller, Mai Tu Xuan, and J.-J. Simond, «Computation of the no-load voltage waveform of laminated salient-pole synchronous generators», *IEEE Trans. Ind. Appl.*, vol. 42, n° 3, p. 681-687, juin 2006.
- [7] «IEEE Standard for Salient-Pole 50 Hz and 60 Hz Synchronous Generators and Generator/Motors for Hydraulic Turbine Applications Rated 5 MVA and Above». 2006.
- [8] A. R. Munoz and T. A. Lipo, «Complex vector model of the squirrel-cage induction machine including instantaneous rotor bar currents », *IEEE Trans. Ind. Appl.*, vol. 35, n° 6, p. 1332-1340, 1999.
- [9] S. Keller, « large low speed hydrogenerators - ump and additional damper losses in eccentricity conditions », Theses, École Polytechnique Fédérale de Lausanne, 2007.
- [10] G. Houdouin, « Contribution à la modélisation de la machine asynchrone en présence de défaut rotorique », Theses, Université du Havre, 2004.