

#### Optimal Design Exploration Using Global Response Surface Method: Rail Crush

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#### 1. Introduction

As design exploration and optimization methods have become commonly accepted across a range of industries, such as aerospace, automotive or oil and gas, they are frequently utilized as standard practice to efficiently produce designs and aid critical engineering decisions. The widespread acceptance of these methods coupled with the power of modern computing has led to applications across a range of design problems and ever-increasing complexity. The size and scope of this expansion continually pushes the boundaries of existing exploration and optimization methods. Furthermore, a complete exploration of the optimal design space includes computationally intensive features such as multi-objective optimization, to understand the trade-off between competing objectives, and global optimization, to avoid local extrema.

Traditional design exploration relies on design of experiments (DOE) and building computationally inexpensive mathematical response surface models which are used as surrogates in the optimization algorithm. The drawback to this technique is that the quality of the optimization is entirely dependent on the predictive quality of the underlying response surface. A poor quality surrogate can produce an optimal design which does not agree with a re-analysis of the original problem using the optimal design variables. One solution to this limitation is to work with adaptive response surfaces, which update the underlying surrogate throughout the optimization process based on periodic evaluations of the original problem. This adaptive solution solves the issue of inaccurate surrogates but is limited to serial analysis of the original problem. Without the parallel analysis of the DOE in the traditional approach, this adaptive method loses computational efficiency. The global response surface method (GRSM) contained in Altair's HyperStudy the concepts of an adaptive response surface based optimization with parallel analysis to provide an accurate and efficient optimization scheme containing global and local search capability along with the capability to handle multi-objective formulations, as well.

The following sections of this paper present an overview of the practical usage of the GRSM algorithm, followed by application problems of rail crush design optimization for a single and multi-objective formulation.

#### 2. GRSM Optimization in HyperStudy

Like most optimization algorithms, GRSM advances in iterations throughout the process. However, the first iteration of GRSM is unique from all subsequent iterations. Within the first iteration, a DOE is constructed internally to provide the data to construct an initial response surface. GSRM uses advanced methods to create response surfaces from a small number of data points, which allows GSRM to remain efficient on problems with large numbers of design variables. Specifically, for N design variables the number of points contained in the initial DOE defaults to take the smaller value of N+2 or 20. The best point, or points in the case of multiple objectives, from this DOE is reported as the iterate(s) of this first iteration. The optimization problem is then solved on the constructed response surface and the optimal design variables' values are used in the next iteration.

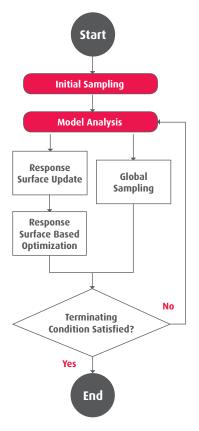


Figure 1. Global response surface method (GRSM) flow-chart

All iterations beyond the initial step follow identical steps. A new DOE is constructed using the optimal points from the previous iteration. This set of data points is supplemented with additional points spread throughout the design space and provide a balance between local and global search. The total number of points contained in this DOE can be controlled by a user parameter. This DOE is executed, the adaptive response surface is updated to absorb the new design points, and the optimization problem is solved again on the newly constructed response surface, with the optimal design feeding forward to the next iteration.

Each iteration contains a set of design points to be analyzed; essentially a DOE. Like all DOE approaches, the analysis of any point is independent of the others, meaning that each point can be analyzed simultaneously to decrease the run time. For most applications, a completely simultaneous analysis of the design points is not possible due to computing resource limitations or a limitation on the availability of licensed software. HyperStudy's multi-execute feature allows user to control the number of simultaneous runs to match their unique situational limitations.

The inclusion of global search features places GRSM into the category of exploratory optimization algorithms. This means the method does not show the numerical convergence characteristics typically observed in other algorithms, such as gradient based schemes. While the optimization problem solved within the iteration is controlled by numerical convergence, this convergence criteria is not used when looking at GRSM as a whole. The same extrema may be obtained in multiple sequential iterations of GRSM. This behavior is different than gradient based methods but typical of exploratory methods. Consequently, the terminating condition of GRSM is instead a resource limitation of the maximum number of evaluations.

## 3. Rail Crush Application: Setup

In order to illustrate the application of GRSM to a practical engineering problem, the following design problem is introduced. A 500 mm long, thin walled rail with a square cross section is fixed at one end, and impacted on the opposite end by a 300 kg rigid body of with an initial velocity of 70 km/s. The initial perimeter and thickness of the cross section is 101.6 mm and 1.0 mm, respectively. The rail is analyzed using the Altair RADIOSS finite element solver, with 1280 shell elements and quarter symmetry to simplify the analysis. Figure 2 shows the initial and final time (t = 20 ms) configurations for the rail.

For optimization, 46 design variables are introduced to modify the basic design of the rail. The thickness of the rail is allowed to vary between 1.0 and 4.0 mm. In addition, nine sections along the rail length are modified using five independent shape variables at each section to alter the cross sections. Figure 3 illustrates the five shapes at the fixed end of the rail. In order to maintain metal forming manufacturability, the perimeter of the cross sections may not deviate from 3.0 mm of the nominal design. The responses of engineering interest from the model are the peak reaction force through the section and the total energy absorbed by the rail.





Figure 2. Initial and final time (t = 20 ms) configurations for the rail.

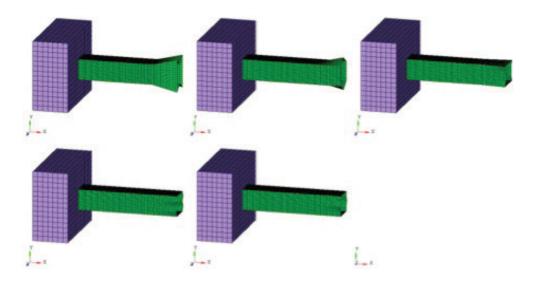


Figure 3. Five shapes at the fixed end of the rail

As discussed in the preceding section, GRSM includes a global search to explore the design space to produce optimal designs. This search is accelerated by introducing additional points for analysis, which can be analyzed simultaneously. With no traditional convergence criteria, the GRSM algorithm only terminates after a prescribed number of evaluations. As with all exploratory methods, any increase in allowed run time is more likely to produce better results, but practical time considerations (such as deadlines or hardware availability) usually restrict the amount of time allotted to find an answer. To this end, the optimization studies in this paper are designed to be limited to an "over-the-weekend" run time. This means launching the job at the end of Friday, with an expectation of completion by the start of work on Monday morning. All of the work contained in this paper, was conducted using an 8 CPU workstation laptop. In order to maximize the efficiency of the GRSM algorithm, a study was conducted to find a balance between the number of simultaneous runs and the number of assigned CPUs for analysis. Two simultaneous jobs with 3 CPUs assigned to each run resulted in the best throughput by completing roughly 2 jobs every 10 minutes. With a total run time target of approximately 60 hours, the number of evaluations was conservatively set to 700. The parameter to control the number of points per iteration is left at the default value of 3. It is noted here that each GRSM run conducted using this methodology did finish over the weekend as intended.



## 4. Rail Crush Application: Multiple Objectives

Early in the design process, the required performance metrics may yet to be finalized, and a crucial piece of information is the trade-off between competing criteria, for example the weight versus stiffness, Multi-objective optimization problems solve this problem by exploring the design space to locate the section of the design space that is Pareto optimal: the idea that one objective cannot be made better without making another worse. These trade-offs can be visualized on the Pareto front, a curve (or surfaces in the case of 3 or more objectives) that shows that trade-off between the competing objectives. The information contained in the Pareto curve is extremely useful to driving preliminary engineering design decisions. For this paper, a multi-objective optimization with two objectives is selected: maximizing the total energy and minimizing the peak internal force. Figure 4 shows the resulting Pareto curve for the two objectives. With only 700 allowable evaluations, the curve contains a relatively small number of non-dominated designs, however the density if is sufficient to observe the basic relationships between the competing objectives. In particular, it is clear that any increase in the total energy will cause an increase in the peak force. The Pareto curve also helps in realistic formulation of the design problem by setting the right design expectations. For example in Figure 4, we see that the Force values are within 50 and 475 kN while total energy values are between 12 and 53 kJ. This tells us that we cannot expect to lower the forces below 50 kN and/or increase the total energy above 53 kJ without altering more than the current design variables. One more important piece of information we can get from the Pareto front is the when the trade-off between objective functions become too compromising. Again, in Figure 4, we can observe that the slope of the curve changes roughly around total energy of 45 kJ. Before this threshold, the slope of the curve is smaller than after this threshold. This means that a unit increase in total energy until 45 kJ leads to less increase in force compared a unit increase in total energy after 45 kJ. As a result, the force performance is more compromised for total energy requirements higher than 45 kJ.

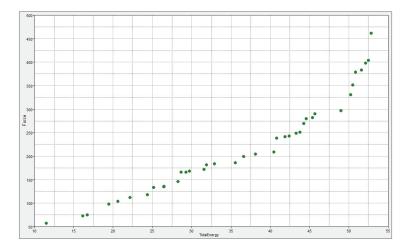


Figure 4. Pareto front

## 5. Rail Crush Application: Single Objective

Single objective optimization is relevant if the set of specific design requirements is known, and a single design response is to be optimized. For the case studied here, we assume the maximum internal force is to be maintained below 150 kN, and the objective is to maximize the absorbed energy. The iteration history of the energy objective is shown in Figure 5. From the plot, it is clear that the optimization does result in a design with much better energy absorption than the nominal design seen in iteration 1. The history also shows an uneven progression of the objective as the optimization progresses as expected from exploratory methods. This step-like objective history is typical with GRSM and shows the importance of the maximum-evaluation-based termination.

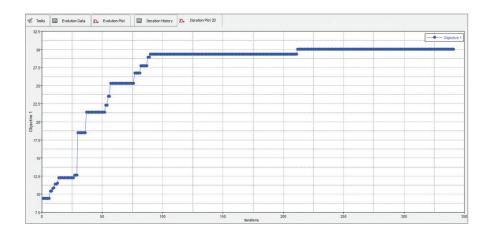


Figure 5. Iteration history

# 6. Summary

As engineering requirements become more sophisticated, the complexity of industry design optimization problems frequently include large numbers of design variables as well as intensive optimal design studies including global search and multi-objective formulations. The Global Response Surface Method (GRSM) within HyperStudy can be applied to these demanding problems. An overview of the GRSM algorithm was presented to clarify the relevant concepts for effective usage. A practical application of GRSM has been demonstrated on the design of an impacted rail, for both single and multiple objective considerations.