



# STEEL DESIGN EXAMPLES TO CSA S16-14 DESIGN OF STRUCTURES

Siri Sasibut – Altair / May 2022

The Steel Design Examples document contains 33 Canadian Code CSA S16-14 steel design examples. These steel design examples are adopted from a subset of the AISC's publication *Design Examples* (version 14.2), with conversion to metric units.

Included with each example are S-FRAME-S-STEEL example files (.TEL files) to illustrate the CSA S16-14 steel design code and to enable the user to verify the accuracy of Altair's steel design program, S-STEEL.

Through design examples, the solution to a given design problem is obtained either by applying relevant clauses in the Standard, or using one or more of the tables in the Handbook, or sometimes both.

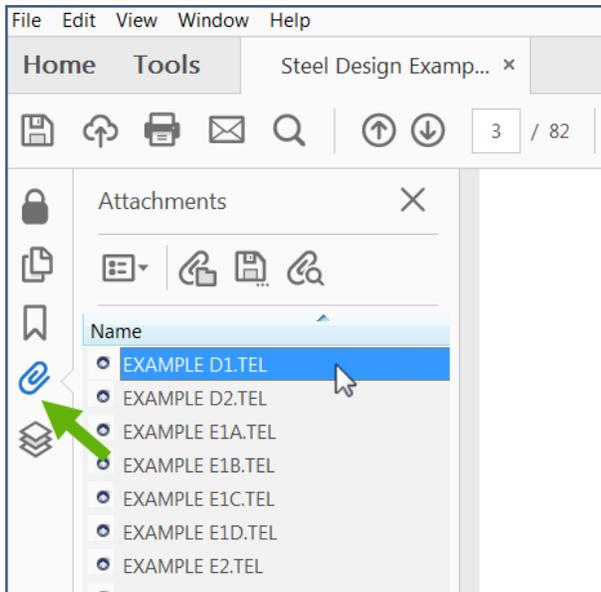
The design example names used in this document are similar to the AISC design code examples, with some minor modifications when appropriate. Users can easily reference and compare both design code examples if required.

While any two counterparts of a design example in both documents are similar in their geometry, support conditions, and loading, they may not be identical due to some round-off or adjusted numbers.

## Accessing the Steel Design Example Files

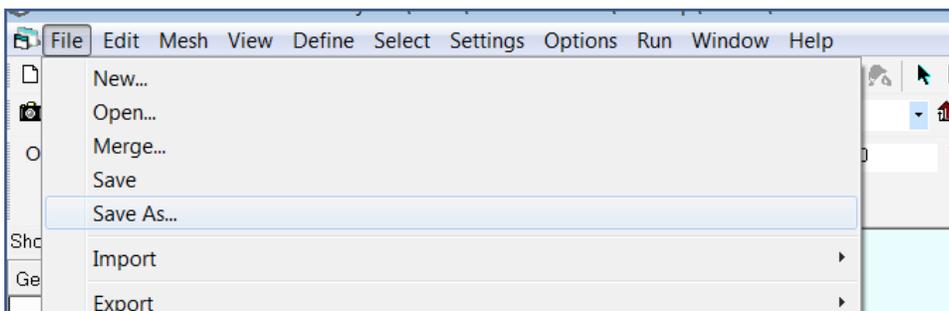
S-FRAME-S-STEEL example files (.tel) are included for each steel design example. A list of all attached files can be found by clicking on the Adobe paperclip icon.

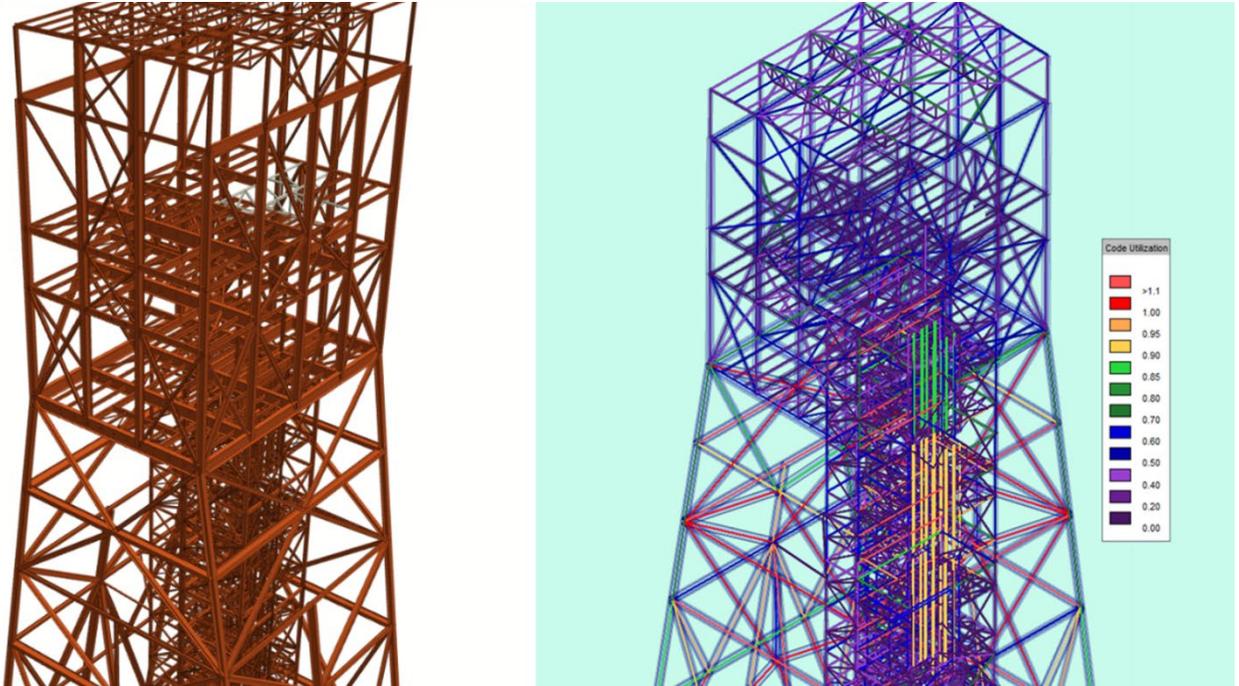
Left-click on any highlighted example file to open – we recommend that you save the file somewhere secure on your machine, prior to running an analysis, and code check.



Once the file has been saved, you can run an analysis and a steel design check to generate results that can be used to validate results with the manual calculations in this guide.

This document does not explicitly describe how to create the S-FRAME models and assumes that the user has familiarity with the software, should they plan to utilize the models.





## CONVENTIONS USED

The following conventions are used throughout these examples:

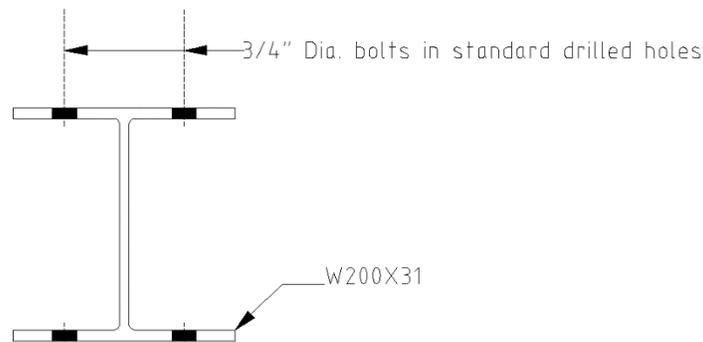
1. The clauses, when being referenced, are from the CSA S16-14 *Design of Steel Structures* standard.
2. The 11<sup>th</sup> Edition of the CISC *Handbook of Steel Construction* is referred to as the *CISC Handbook*.
3. The effects (forces and/or moments) in a member under factored loads are determined according to the requirements of *National Building Code of Canada 2015*, NBCC 2015.
4. Each design example is accompanied with an S-FRAME model file, which contains pertinent analysis and design data of the design problem, with the same name, e.g., *Example F1-1A.TEL*.
5. The version of document reflects that of S-FRAME/S-STEEL which is used to obtain the analysis and design results of the design problems.

## Chapter D Design of Members for Tension

### EXAMPLE D.1 W-SHAPE TENSION MEMBER

#### Given:

Select a 200mm W-shape, CSA G40.21- 350W, to carry a dead load of 130 kN and a live load of 400 kN in tension. The member is 7.5m long. Verify the member resistance under the factored force with the bolted end connection shown. Verify that the member satisfies the maximum slenderness ratio. Assume the connection limit states do not govern.



#### Solution:

From Division B Part 4 of NBCC 2015, the factored tensile force is:

$$T_f = 1.25D + 1.5L = 1.25 \times 130 + 1.5 \times 400 = 762.5 \text{ kN}$$

Try a W200x31.

From CISC *Handbook* Table 6-3, the material properties are as follows:

W200x31

CSA G40.21 – 350W

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

W200x31

$$A_g = 3970 \text{ mm}^2$$

$$b = 134 \text{ mm}$$

$$t = 10.2 \text{ mm}$$

$$d = 210 \text{ mm}$$

$$r_y = 32 \text{ mm}$$

#### Tensile Yielding

From Clause 13.2.a), the tensile yielding is the least of  $T_r = \phi A_g F_y$  and  $T_r = \phi_u A_{ne} F_u$ .

From Clause 13.1. a),  $\phi = 0.90$  and  $\phi_u = 0.75$ .

$$\begin{aligned} T_r &= \phi A_g F_y \\ &= 0.90 \times 3970 \text{mm}^2 \times 350 \text{MPa} \\ &= 1250 \text{ kN} \end{aligned}$$

### *Tensile Rupture*

Calculate  $A_{ne}$  using Clause 12.3.3.1 and Clause 12.3.1.a)

$$\begin{aligned} A_{ne} &= A_n \\ &= w_n t \\ &= 3970 \text{mm}^2 - 4 \times 21 \text{mm} \times 10.2 \text{mm} \\ &= 3113 \text{ mm}^2 \end{aligned}$$

The tensile rupture resistance is,

$$\begin{aligned} T_r &= \phi_u A_{ne} F_u \\ &= 0.75 \times 3113 \text{mm}^2 \times 450 \text{MPa} \\ &= 1050 \text{ kN (governs)}. \end{aligned}$$

Since  $T_r$  is the least of  $T_r = \phi A_g F_y$  and  $T_r = \phi_u A_{ne} F_u$ , in this case  $T_r = 1050 \text{ kN}$

$$T_f = 762.5 \text{ kN} < T_r \quad \mathbf{O.K.}$$

### *Check Slenderness Ratio*

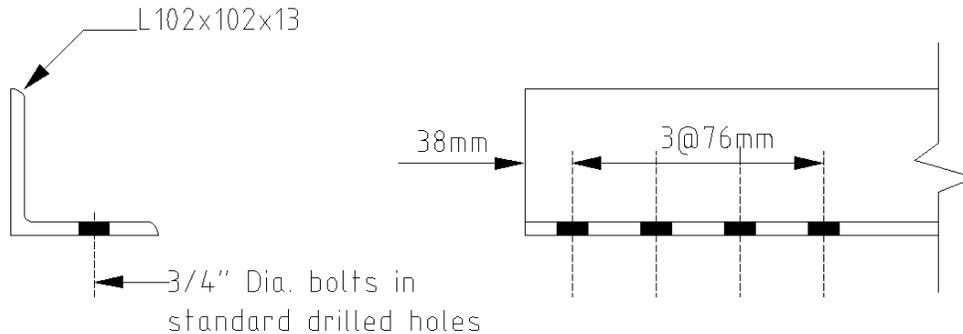
$$\begin{aligned} \frac{L}{r_y} &= \frac{7500 \text{mm}}{32 \text{mm}} \\ &= 234 < 300 \text{ from Clause 10.4.2.2} \quad \mathbf{O.K.} \end{aligned}$$

$$T_r (\mathbf{S - STEEL}) = 1074 \text{ kN (2.29 \% Diff.)}$$

## EXAMPLE D.2 SINGLE ANGLE TENSION MEMBER

### Given:

Verify the factored tensile resistance of an L 102x102x13, CSA G40.21- 300W, with one line of (4)  $\frac{3}{4}$ -in.-diameter bolts in standard holes. The member carries a dead load of 90 kN and a live load of 270 kN in tension. Calculate at what length this tension member would cease to satisfy the maximum slenderness ratio. Assume that connection limit states do not govern.



### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

$$\begin{aligned} & \text{L } 102 \times 102 \times 13 \\ & \text{CSA G40.21- 300W} \\ & F_y = 300 \text{ MPa} \\ & F_u = 450 \text{ MPa} \end{aligned}$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

$$\begin{aligned} & \text{L } 102 \times 102 \times 13 \\ & A_g = 2420 \text{ mm}^2 \\ & r'_y = 19.9 \text{ mm} \end{aligned}$$

From Division B Part 4 of NBCC 2015, the factored tensile force is:

$$T_f = 1.25D + 1.5L = 1.25 \times 90 + 1.5 \times 270 = 518 \text{ kN}$$

### Tensile Yielding

From Clause 13.2.a), the tensile yielding is the least of  $T_r = \phi A_g F_y$  and  $T_r = \phi_u A_{ne} F_u$ .

From Clause 13.1. a),  $\phi = 0.90$  and  $\phi_u = 0.75$ .

$$\begin{aligned} T_r &= \phi A_g F_y \\ &= 0.90 \times 2420 \text{ mm}^2 \times 300 \text{ MPa} \end{aligned}$$

$$= 653 \text{ kN.}$$

### Tensile Rupture

Calculate  $A_{ne}$  using Clause 12.3.3.2.b(i) and Clause 12.3.1.a)

$$\begin{aligned} A_{ne} &= 0.80 A_n \\ &= 0.80 w_n t \\ &= 0.80(2420\text{mm}^2 - 21\text{mm} \times 13\text{mm}) \\ &= 1717\text{mm}^2 \end{aligned}$$

The tensile rupture resistance is,

$$\begin{aligned} T_r &= \phi_u A_{ne} F_u \\ &= 0.75 \times 1717\text{mm}^2 \times 450\text{MPa} \\ &= 579.5 \text{ kN (governs)} \end{aligned}$$

The  $\perp$  102x102x13 factored tensile resistance is governed by the tensile rupture limit state.

$$T_f = 518 \text{ kN} < T_r \quad \text{O.K.}$$

$$T_r (\text{S - STEEL}) = 586\text{kN} (1.12 \% \text{ Diff.})$$

### $L_{max}$

According to Clause 10.4.2.2, the maximum slenderness ratio is 300.

$$\begin{aligned} L_{max} &= 300 r_y' \\ &= 300 \times 19.9\text{mm} \\ &= 5970 \text{ mm} \end{aligned}$$

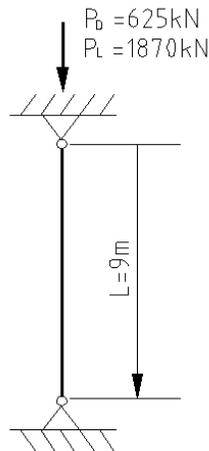
Note: The L/r limit may be waived if other means are provided to control flexibility, sag, vibration and slack in a manner commensurate with the service conditions of the structure.

## Chapter E Design of Members for Compression

### EXAMPLE E.1A W-SHAPE COLUMN DESIGN WITH PINNED ENDS

#### Given:

Select a ASTM A992 ( $F_y = 345 \text{ Mpa}$ ) W-shape column, to carry an axial dead load of 625 kN and live load of 1870 kN. The column is 9m long and is pinned top and bottom in both axes. Limit the column size to a nominal 360mm shape.



#### Solution:

From Division B Part 4 of NBCC 2015, the factored axial compressive force is:

$$C_f = 1.25D + 1.5L = 1.25 \times 625 + 1.5 \times 1870 = 3586 \text{ kN}$$

#### Column Selection

From CISC *Handbook* Annex F Clause F.2, for a pinned-pinned condition,  $K = 1.0$ .

Because the unbraced length is the same in both the x-x and y-y directions and  $r_x$  exceeds  $r_y$  for all W-shapes, y-y axis buckling will govern.

Search the table with an effective length,  $KL = 9000 \text{ mm}$ , and proceed across the table in CISC *Handbook* Part 4 until reaching the least weight shape with a factored compressive resistance that equals or exceeds the factored axial compressive force. Select a W360x196.

From the table in CISC *Handbook* Part 4, the factored compressive resistance for a y-y axis effective length of 9000 mm is:

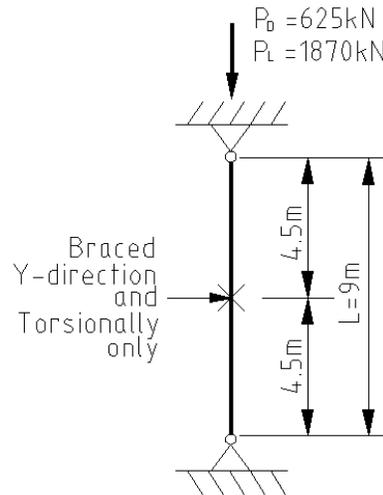
$$C_r = 3610\text{kN} > 3586\text{kN} \quad \text{O.K.}$$

$$C_r (\mathbf{S} - \mathbf{STEEL}) = 3608\text{kN} (0.05 \% \text{ Diff.})$$

## EXAMPLE E.1B W-SHAPE COLUMN DESIGN WITH INTERMEDIATE BRACING

### Given:

Redesign the column from Example E.1A assuming the column is laterally braced about the y-y axis and torsionally braced at the midpoint.



### Solution:

From Division B Part 4 of NBCC 2015, the factored axial compressive force is:

$$C_f = 1.25D + 1.5L = 1.25 \times 625 + 1.5 \times 1870 = 3586 \text{ kN}$$

### Column Selection

From CISC *Handbook* Annex F Clause F.2, for a pinned-pinned condition,  $K = 1.0$ .

Because the unbraced lengths differ in the two axes, select the member using the y-y axis then verify the strength in the x-x axis.

Search the table with an effective length,  $KL_y = 4500 \text{ mm}$ , and proceed across the table in CISC *Handbook* Part 4 until reaching the least weight shape with a factored compressive resistance that equals or exceeds the factored axial compressive force. Try a W360x134. A 4500mm long W360x134 provides a factored compressive resistance in the y-y direction of 4370kN, and  $\frac{r_x}{r_y} = 1.66$ .

As for x-x axis, the effective length  $KL_x = 9000\text{mm}$ . The equivalent y-y axis effective length for strong axis buckling is computed as:

$$KL = \frac{9000}{1.66} = 5422 \text{ mm}(\text{governs}) > KL_y = 4500 \text{ mm}$$

The factored compressive resistance is governed by the x-x axis flexural buckling limit state.

The W360x134 has a factored compressive resistance of 3947 kN with an effective length of  $KL = 5422\text{mm}$ .

$$C_r = 3947 \text{ kN} > C_f = 3586 \text{ kN} \quad \text{O.K.}$$

$$C_r (\text{S - STEEL}) = 3449\text{kN} (12.62 \% \text{ Diff.})$$

**Note:** This difference is mainly due to S-STEEL conservatively assuming  $KL_z = \text{the greater of } KL_x \text{ and } KL_y = 9000\text{mm}$ .

The factored compressive resistance of the columns described in Examples E.1A and E.1B are easily selected directly from the CISC *Handbook* Tables. The factored compressive resistance can also be verified by hand calculations, as shown in the following Examples E.1C and E.1D.

## EXAMPLE E.1C W-SHAPE FACTORED RESISTANCE CALCULATION

### Given:

Calculate the factored resistance of a W360x196 column with unbraced lengths of 9m in both axes. The material properties and loads are as given in Example E.1A.

### Solution:

From CISC *Handbook* Table 6-7, the material properties are as follows:

ASTM A992

$$F_y = 345 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

W360x196

$$A_g = 25000 \text{ mm}^2$$

$$r_x = 159 \text{ mm}$$

$$r_y = 95.6 \text{ mm}$$

### Slenderness Check

From CISC *Handbook* Annex F Clause F.2, for a pinned-pinned condition,  $K = 1.0$ .

Because the unbraced length is the same for both axes, the y-y axis will govern.

$$\frac{KL}{r_y} = \frac{1.0 \times 9000}{95.6} = 94$$

For  $F_y = 345 \text{ MPa}$ , the unit factored compressive resistance,  $C_r/A = 144 \text{ MPa}$  is interpolated from CISC *Handbook* Table 4-3.

Therefore,  $C_r = 144 \text{ MPa} \times 25000 \text{ mm}^2 = 3600 \text{ kN} > 3586 \text{ kN}$  **O.K.**

$C_r$  (S – STEEL) = 3608kN (0.02 % Diff.)

Note that the calculated value is approximately equal to the tabulated value.

## EXAMPLE E.1D W-SHAPE FACTORED RESISTANCE CALCULATION

### Given:

Calculate the factored resistance of a W360x134 with a strong axis unbraced length of 9m and weak axis and torsional unbraced lengths of 4.5m. The material properties and loads are as given in Example E.1A.

### Solution:

From CISC *Handbook* Table 6-7, the material properties are as follows:

ASTM A992

$$F_y = 345 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

W360x134

$$A_g = 17100 \text{ mm}^2$$

$$r_x = 156 \text{ mm}$$

$$r_y = 94 \text{ mm}$$

### Slenderness Check

From CISC *Handbook* Annex F Clause F.2, for a pinned-pinned condition,  $K = 1.0$ .

$$\frac{KL_x}{r_x} = \frac{1.0 \times 9000}{156} = 58 \text{ (governs)}$$

$$\frac{KL_y}{r_y} = \frac{1.0 \times 4500}{94} = 48$$

### Elastic Buckling Stress

The unit factored compressive resistance may be interpolated from CISC *Handbook* Table 4-3 or calculated directly from Clause 13.3.1 as follows:

Calculate the elastic buckling stress,  $F_e$ .

$$F_e = \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2}$$

$$= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(58)^2}$$

$$= 587 \text{ MPa}$$

$$\lambda = \sqrt{\frac{F_y}{F_e}}$$

$$= \sqrt{\frac{345}{587}}$$

$$= 0.767$$

### Factored Axial Compressive Resistance

For a ASTM A992 W360x134, it meets the requirements of CSA S16-14 Table 1. For fabricated structural sections,  $n=1.34$ .

$$C_r = \frac{\phi A F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}}$$

$$= \frac{0.9 \times 17100 \text{ mm}^2 \times 345 \text{ MPa}}{(1 + 0.767^{2 \times 1.34})^{\frac{1}{1.34}}}$$

$$= 3940 \text{ kN} > 3586 \text{ kN} \quad \mathbf{O.K.}$$

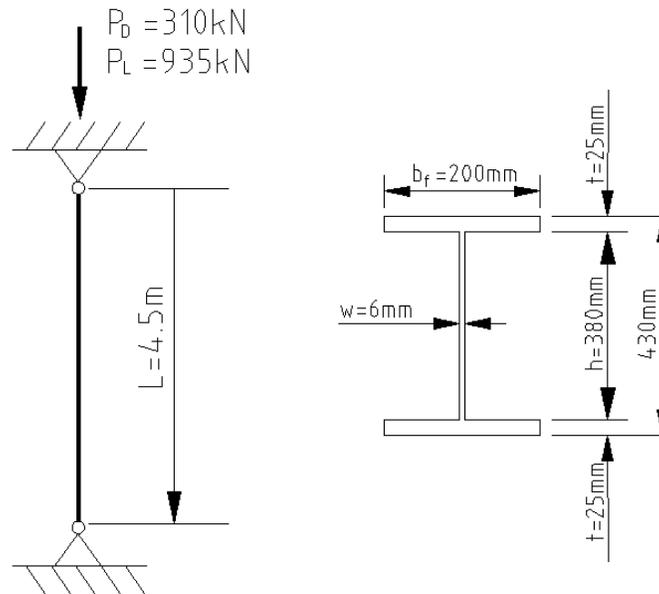
$$C_r (\mathbf{S - STEEL}) = 3449 \text{ kN} (12.46 \% \text{ Diff.})$$

**Note:** This difference is mainly due to S-STEEL conservatively assuming  $KL_z = \text{the greater of } KL_x \text{ and } KL_y = 9000 \text{ mm}$ .

## EXAMPLE E.2 BUILT-UP COLUMN WITH A SLENDER WEB

### Given:

Verify that a built-up, CSA G40.21- 350W column with PL25mm×200mm flanges and a PL6mm×380mm web is sufficient to carry a dead load of 310 kN and live load of 935 kN in axial compression. The column length is 4.50 m and the ends are pinned in both axes.



### Solution:

From CISC *Handbook* Table 6-7, the material properties are as follows:

Built-Up Column  
 CSA G40.21- 350W  
 $F_y = 350\text{ MPa}$   
 $F_u = 450\text{ MPa}$

The geometric properties are as follows:

Built-Up Column  
 $d = 430\text{ mm}$   
 $b_f = 200\text{ mm}$   
 $t = 25\text{ mm}$   
 $h = 380\text{ mm}$   
 $w = 6\text{ mm}$

From Division B Part 4 of NBCC 2015, the factored axial compressive force is:

$$C_f = 1.25D + 1.5L = 1.25 \times 310 + 1.5 \times 935 = 1790 \text{ kN}$$

*Built-Up Section Properties (ignoring fillet welds)*

$$\begin{aligned} A &= 2(25\text{mm})(200\text{mm}) + (6\text{mm})(380\text{mm}) \\ &= 12280 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} I_y &= \frac{2(25\text{mm})(200\text{mm})^3}{12} + \frac{(380\text{mm})(6\text{mm})^3}{12} \\ &= 33340173 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} r_y &= \sqrt{\frac{I_y}{A}} \\ &= \sqrt{\frac{33340173 \text{ mm}^4}{12280 \text{ mm}^2}} \\ &= 52 \text{ mm} \end{aligned}$$

$$\begin{aligned} I_x &= \sum A d^2 + \sum \frac{bh^3}{12} \\ &= 2(25\text{mm} \times 200\text{mm})(202.5\text{mm})^2 + \frac{2(200\text{mm})(25\text{mm})^3}{12} + \frac{(6\text{mm})(380\text{mm})^3}{12} \\ &= 4.38 \times 10^8 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} r_x &= \sqrt{\frac{I_x}{A}} \\ &= \sqrt{\frac{4.38 \times 10^8 \text{ mm}^4}{12280 \text{ mm}^2}} \\ &= 189 \text{ mm} \end{aligned}$$

$$\begin{aligned} \bar{r}_o^2 &= x_o^2 + y_o^2 + r_x^2 + r_y^2 \\ &= 0 + 0 + (189\text{mm})^2 + (52\text{mm})^2 \\ &= 38425 \text{ mm}^2 \end{aligned}$$

*Elastic Flexural Buckling Stress*

From CISC *Handbook* Annex F Clause F.2, for a pinned-pinned condition,  $K = 1.0$ .

Because the unbraced length is the same for both axes, the y-y axis will govern by inspection.

$$\frac{KL_y}{r_y} = \frac{1.0 \times 4500}{52} = 86.5$$

$$\begin{aligned}
 F_e &= \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} && \text{(Clause 13.3.1)} \\
 &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(86.5)^2} \\
 &= 264 \text{ MPa}
 \end{aligned}$$

#### Elastic Torsional Buckling Stress

Note: Torsional buckling generally will not govern if  $KL_y \geq KL_z$ ; however, the check is included here to illustrate the calculation.

From CISC *Handbook* Section 7,

$$\begin{aligned}
 C_w &= \frac{1}{24} (d - t)^2 b^3 t \\
 &= \frac{1}{24} (430 \text{ mm} - 25 \text{ mm})^2 (200 \text{ mm})^3 (25 \text{ mm}) \\
 &= 1.367 \times 10^{12} \text{ mm}^6
 \end{aligned}$$

$$\begin{aligned}
 J &= \frac{1}{3} [2bt^3 + (d - t)w^3] \\
 &= \frac{1}{3} [2(200 \text{ mm})(25 \text{ mm})^3 + (430 \text{ mm} - 25 \text{ mm}) \times (6 \text{ mm})^3] \\
 &= 2112493 \text{ mm}^4
 \end{aligned}$$

From Clause 13.3.2.a), the factored compressive resistance for doubly symmetric sections is the least of  $F_{ex}$ ,  $F_{ey}$ , and  $F_{ez}$ .

$$\frac{KL_x}{r_x} = \frac{1.0 \times 4500}{189} = 23.8$$

$$\begin{aligned}
 F_{ex} &= \frac{\pi^2 E}{\left(\frac{KL_x}{r_x}\right)^2} \\
 &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(23.8)^2}
 \end{aligned}$$

$$= 3485 \text{ MPa}$$

$$\begin{aligned}
 F_{ey} &= \frac{\pi^2 E}{\left(\frac{KL_y}{r_y}\right)^2} \\
 &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(86.5)^2} \\
 &= 264 \text{ MPa} \quad (\text{governs})
 \end{aligned}$$

$$\begin{aligned}
 F_{ez} &= \left( \frac{\pi^2 EC_w}{(KL_z)^2} + GJ \right) \frac{1}{Ar_o^2} \\
 &= \left( \frac{\pi^2 (200 \times 10^3 \text{ MPa})(1.367 \times 10^{12} \text{ mm}^6)}{(1 \times 4500 \text{ mm})^2} + (75 \times 10^3 \text{ MPa})(2112493 \text{ mm}^4) \right) \\
 &\quad \times \frac{1}{(12280 \text{ mm}^2)(38425 \text{ mm}^2)} \\
 &= 618 \text{ MPa}
 \end{aligned}$$

So  $F_e = 264 \text{ MPa}$ , the flexural buckling limit state controls.

#### Factored Axial Compressive Resistance

For this CSA G40.21- 350W built-up column,  $\frac{b_{el}}{t} = \frac{100 \text{ mm}}{25 \text{ mm}} = 4 < \frac{200}{\sqrt{F_y}} = \frac{200}{\sqrt{350 \text{ MPa}}} = 10.7$ ,  $\frac{h}{w} = \frac{380}{6} = 63 > \frac{670}{\sqrt{F_y}} = \frac{670}{\sqrt{350 \text{ MPa}}} = 35.8$ , which exceeds the requirements in CSA S16-14 Table 1. Clause 13.3.5(a) shall be applied in this situation, where  $n=2.24$ .

#### Slenderness

$$\begin{aligned}
 \lambda &= \sqrt{\frac{F_y}{F_e}} \\
 &= \sqrt{\frac{350}{264}} \\
 &= 1.15
 \end{aligned}$$

$A_e$  is the effective area, with the reduced element width meeting the requirement in CSA S16-14 Table1.

$$\begin{aligned}h_e &= \frac{670}{\sqrt{F_y}} \times w \\ &= 35.8 \times 6\text{mm} \\ &= 214\text{ mm}\end{aligned}$$

$$\begin{aligned}A_e &= 2(25\text{mm})(200\text{mm}) + (6\text{mm})(214\text{mm}) \\ &= 11284\text{ mm}^2\end{aligned}$$

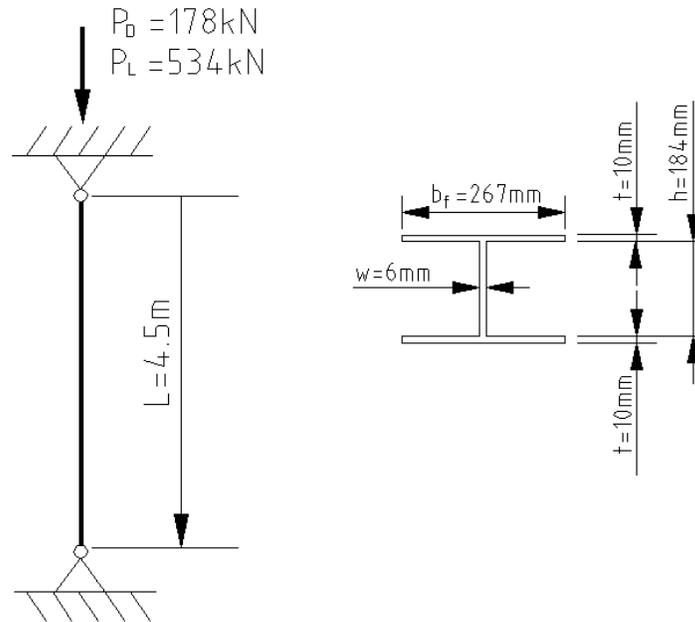
$$\begin{aligned}C_r &= \frac{\phi A_e F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}} \\ &= \frac{0.9 \times 11284\text{mm}^2 \times 350\text{MPa}}{(1 + 1.15^{2 \times 2.24})^{\frac{1}{2.24}}} \\ &= 2220\text{ kN} > 1790\text{ kN} \quad \mathbf{O.K.}\end{aligned}$$

$$C_r (\mathbf{S - STEEL}) = 2221\text{kN} (0.05\% \text{ Diff.})$$

### EXAMPLE E.3 BUILT-UP COLUMN WITH SLENDER FLANGES

#### Given:

Determine if a built-up, CSA G40.21- 350W column with PL10mm×267mm flanges and a PL6mm×184mm web is sufficient to carry a dead load of 178 kN and live load of 534 kN in axial compression. The column length is 4.50 m and the ends are pinned in both axes.



#### Solution:

From CISC *Handbook* Table 6-7, the material properties are as follows:

Built-Up Column  
 CSA G40.21- 350W  
 $F_y = 350\text{ MPa}$   
 $F_u = 450\text{ MPa}$

The geometric properties are as follows:

Built-Up Column  
 $d = 204\text{ mm}$   
 $b_f = 267\text{ mm}$   
 $t = 10\text{ mm}$   
 $h = 184\text{ mm}$   
 $w = 6\text{ mm}$

From Division B Part 4 of NBCC 2015, the factored axial compressive force is:

$$C_f = 1.25D + 1.5L = 1.25 \times 178 + 1.5 \times 534 = 1023.5 \text{ kN}$$

*Built-Up Section Properties (ignoring fillet welds)*

$$\begin{aligned} A &= 2(10\text{mm})(267\text{mm}) + (6\text{mm})(184\text{mm}) \\ &= 6444 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} I_y &= \frac{2(10\text{mm})(267\text{mm})^3}{12} + \frac{(184\text{mm})(6\text{mm})^3}{12} \\ &= 31726917 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} r_y &= \sqrt{\frac{I_y}{A}} \\ &= \sqrt{\frac{31726917 \text{ mm}^4}{6444 \text{ mm}^2}} \\ &= 70 \text{ mm} \end{aligned}$$

$$\begin{aligned} I_x &= \sum A d^2 + \sum \frac{bh^3}{12} \\ &= 2(10\text{mm} \times 267\text{mm})(97\text{mm})^2 + \frac{2(267\text{mm})(10\text{mm})^3}{12} + \frac{(6\text{mm})(184\text{mm})^3}{12} \\ &= 5.34 \times 10^7 \text{ mm}^4 \end{aligned}$$

$$\begin{aligned} r_x &= \sqrt{\frac{I_x}{A}} \\ &= \sqrt{\frac{5.34 \times 10^7 \text{ mm}^4}{6444 \text{ mm}^2}} \\ &= 91 \text{ mm} \end{aligned}$$

$$\begin{aligned} \bar{r}_o^2 &= x_o^2 + y_o^2 + r_x^2 + r_y^2 \\ &= 0 + 0 + (91\text{mm})^2 + (70\text{mm})^2 \\ &= 13181 \text{ mm}^2 \end{aligned}$$

*Elastic Flexural Buckling Stress*

From CISC *Handbook* Annex F Clause F.2, for a pinned-pinned condition,  $K = 1.0$ .

Because the unbraced length is the same for both axes, the y-y axis will govern by inspection.

$$\frac{KL_y}{r_y} = \frac{1.0 \times 4500}{70} = 64.3$$

$$\begin{aligned}
 F_e &= \frac{\pi^2 E}{\left(\frac{KL}{r}\right)^2} && \text{(Clause 13.3.1)} \\
 &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(64.3)^2} \\
 &= 477 \text{ MPa}
 \end{aligned}$$

#### *Elastic Torsional Buckling Stress*

Note: Torsional buckling generally will not govern if  $KL_y \geq KL_z$ ; however, the check is included here to illustrate the calculation.

From CISC *Handbook* Part 7,

$$\begin{aligned}
 C_w &= \frac{1}{24} (d - t)^2 b^3 t \\
 &= \frac{1}{24} (204 \text{ mm} - 10 \text{ mm})^2 (267 \text{ mm})^3 (10 \text{ mm}) \\
 &= 2.98 \times 10^{11} \text{ mm}^6
 \end{aligned}$$

$$\begin{aligned}
 J &= \frac{1}{3} [2bt^3 + (d - t)w^3] \\
 &= \frac{1}{3} [2(267 \text{ mm})(10 \text{ mm})^3 + (204 \text{ mm} - 10 \text{ mm}) \times (6 \text{ mm})^3] \\
 &= 191968 \text{ mm}^4
 \end{aligned}$$

From Clause 13.3.2.a), the factored compressive resistance for doubly symmetric sections is the least of  $F_{ex}$ ,  $F_{ey}$ , and  $F_{ez}$ .

$$\frac{KL_x}{r_x} = \frac{1.0 \times 4500}{91} = 49.5$$

$$\begin{aligned}
 F_{ex} &= \frac{\pi^2 E}{\left(\frac{KL_x}{r_x}\right)^2} \\
 &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(49.5)^2} \\
 &= 805 \text{ MPa}
 \end{aligned}$$

$$\begin{aligned}
 F_{ey} &= \frac{\pi^2 E}{\left(\frac{KL_y}{r_y}\right)^2} \\
 &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(64.3)^2} \\
 &= 477 \text{ MPa} \quad \text{(governs)}
 \end{aligned}$$

$$\begin{aligned}
 F_{ez} &= \left( \frac{\pi^2 EC_w}{(KL_z)^2} + GJ \right) \frac{1}{A\bar{r}_o^2} \\
 &= \left( \frac{\pi^2 (200 \times 10^3 \text{ MPa})(2.98 \times 10^{11} \text{ mm}^6)}{(1 \times 4500 \text{ mm})^2} + (75 \times 10^3 \text{ MPa})(191968 \text{ mm}^4) \right) \\
 &\quad \times \frac{1}{(6444 \text{ mm}^2)(13181 \text{ mm}^2)} \\
 &= 511 \text{ MPa}
 \end{aligned}$$

So  $F_e = 477 \text{ MPa}$ , the flexural buckling limit state controls.

#### Factored Axial Compressive Resistance

For this CSA G40.21- 350W built-up column,  $\frac{b_{el}}{t} = \frac{133.5 \text{ mm}}{10 \text{ mm}} = 13.35 > \frac{200}{\sqrt{F_y}} = \frac{200}{\sqrt{350 \text{ MPa}}} = 10.7$ ,  $\frac{h}{w} = \frac{184}{6} = 30.7 <$

$\frac{670}{\sqrt{F_y}} = \frac{670}{\sqrt{350 \text{ MPa}}} = 35.8$ , which exceeds the requirements in CSA S16-14 Table 1. Clause 13.3.5(a) shall be applied in this

situation, where  $n=2.24$ .

#### Slenderness

$$\begin{aligned}
 \lambda &= \sqrt{\frac{F_y}{F_e}} \\
 &= \sqrt{\frac{350}{477}} \\
 &= 0.86
 \end{aligned}$$

$A_e$  is the effective area, with the reduced element width meeting the requirement in CSA S16-14 Table1.

$$\begin{aligned}
 b_{el} &= \frac{200}{\sqrt{F_y}} \times t \\
 &= 10.7 \times 10 \text{ mm}
 \end{aligned}$$

$$= 107 \text{ mm}$$

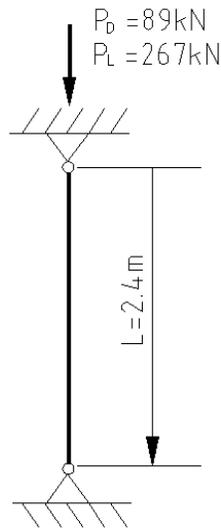
$$\begin{aligned} A_e &= 2(10\text{mm})(214\text{mm}) + (6\text{mm})(184\text{mm}) \\ &= 5384 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} C_r &= \frac{\phi A_e F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}} \\ &= \frac{0.9 \times 5384 \text{ mm}^2 \times 350 \text{ MPa}}{(1 + 0.86^{2 \times 2.24})^{\frac{1}{2.24}}} \\ &= 1411 \text{ kN} > 1023.5 \text{ kN} \quad \mathbf{O. K.} \end{aligned}$$

$$C_r (\mathbf{S - STEEL}) = 1417 \text{ kN} (0.42 \% \text{ Diff.})$$

**EXAMPLE E.5 DOUBLE ANGLE COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS**
**Given:**

Verify the resistance of a 2L 102x89x9.5 LLBB (20mm separation) strut, CSA G40.21- 350W, with a length of 2.4m and pinned ends carrying an axial dead load of 89kN and live load of 267kN. Also, calculate the required number of pretensioned bolted or welded intermediate connectors.


**Solution:**

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 350W

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

L 102x89x9.5 LLBB

$$t = 9.53 \text{ mm}$$

$$y = 30.8 \text{ mm}$$

$$r_z = 18.5 \text{ mm}$$

2L 102x89x9.5 LLBB

$$r_x = 31.9 \text{ mm}$$

$$r_y = 39.7 \text{ mm for } 10\text{mm separation}$$

$$r_y = 43.5 \text{ mm for } 20\text{mm separation}$$

$$A = 3440 \text{ mm}^2$$

From Division B Part 4 of NBCC 2015, the factored compressive force is:

$$C_f = 1.25D + 1.5L = 1.25 \times 89 + 1.5 \times 267 = 512 \text{ kN}$$

*Class of Section*

Width-to-thickness ratios, based on S16-14 Clause 11.2, are as follows:

$$\frac{d}{t} = \frac{102}{9.53} = 10.7 < \frac{250}{\sqrt{F_y}} = 13.4$$

$$\frac{b}{t} = \frac{88.9}{9.53} = 9.3 < \frac{250}{\sqrt{F_y}} = 13.4$$

The angle does not have a slender element.

*Compressive Resistance About Axis X-X, Flexural Mode*

From Clause 13.3.1, the elastic buckling stress  $F_e$  shall be as follows:

$$\frac{KL_x}{r_x} = \frac{1.0 \times 2400}{31.9} = 75$$

$$\begin{aligned} F_e &= \frac{\pi^2 E}{\left(\frac{KL_x}{r_x}\right)^2} \\ &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(75)^2} \\ &= 351 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \lambda &= \sqrt{\frac{F_y}{F_e}} \\ &= \sqrt{\frac{350}{351}} \\ &= 0.998 \end{aligned}$$

$$\begin{aligned} C_{rx} &= \frac{\phi A F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}} \\ &= \frac{0.9 \times 3440 \text{ mm}^2 \times 350 \text{ MPa}}{(1 + 0.998^{2 \times 1.34})^{\frac{1}{1.34}}} \\ &= 647 \text{ kN} > 512 \text{ kN} \end{aligned}$$

### Compressive Resistance About Axis Y-Y, Torsional-Flexural Mode

Based on Clause 13.3.2, the shear center location is as follows:

$$x_o = 0 \quad y_o = y - \frac{t}{2} = 30.8 - \frac{9.53}{2} = 26 \text{ mm}$$

Torsional-flexural section properties are calculated as below.

$$\begin{aligned} \bar{r}_o^2 &= x_o^2 + y_o^2 + r_x^2 + r_y^2 \\ &= 0 + (26\text{mm})^2 + (31.9\text{mm})^2 + (43.5\text{mm})^2 \\ &= 3586 \text{ mm}^2 \end{aligned}$$

$$\begin{aligned} \Omega &= 1 - \left( \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \right) \\ &= 1 - \left( \frac{0^2 + 26^2}{3586} \right) \\ &= 0.81 \end{aligned}$$

Based on Clause 19.2.4(b), the slenderness ratio of the built-up member shall be as follows:

$$\rho_o = \frac{KL_y}{r_y} = \frac{1 \times 2400}{43.5} = 55.2$$

Equivalent slenderness ratio of a component angle, with two welded intermediate connectors spaced at  $L/3 = 2400/3 = 800 \text{ mm}$  and  $K = 0.65$ , is calculated as follows:

$$\rho_i = \frac{KL_z}{r_z} = \frac{0.65 \times 800}{18.5} = 28.1$$

$$\rho_e = \sqrt{\rho_o^2 + \rho_i^2} = \sqrt{55.2^2 + 28.1^2} = 61.9$$

### Elastic Torsional Buckling Stress

From Clause 13.3.2.b), the factored compressive resistance for singly symmetric sections, with the y-axis taken as the axis of symmetry, is the lesser of  $F_{ex}$  and  $F_{ey}$ .

$$\begin{aligned} F_{ey} &= \frac{\pi^2 E}{\rho_e^2} \\ &= \frac{\pi^2 \times 200 \times 10^3}{61.9^2} \\ &= 515.2 \text{ MPa} \end{aligned}$$

$$\begin{aligned}
 F_{ez} &= \left( \frac{\pi^2 EC_w}{(KL_z)^2} + GJ \right) \frac{1}{A\bar{r}_o^2} \\
 &= \left( \frac{\pi^2 (200 \times 10^3 \text{ MPa})(72.8 \times 10^6 \text{ mm}^6)}{(1 \times 2400 \text{ mm})^2} + (77 \times 10^3 \text{ MPa})(104.6 \times 10^3 \text{ mm}^4) \right) \\
 &\quad \times \frac{1}{(3440 \text{ mm}^2)(3586 \text{ mm}^2)} \\
 &= 655 \text{ MPa}
 \end{aligned}$$

$$\begin{aligned}
 F_e &= F_{eyz} = \frac{F_{ey} + F_{ez}}{2\Omega} \left( 1 - \sqrt{1 - \frac{4F_{ey}F_{ez}\Omega}{(F_{ey} + F_{ez})^2}} \right) \\
 &= \frac{515.2 + 655}{2 \times 0.81} \left( 1 - \sqrt{1 - \frac{4 \times 515.2 \times 655 \times 0.81}{(515.2 + 655)^2}} \right) \\
 &= 398 \text{ MPa}
 \end{aligned}$$

*Factored Compressive Resistance*

By comparison, the flexural mode of buckling about axis X-X is governing,  $F_e = 351 \text{ MPa}$ .

$$C_r = C_{rx} = 647 \text{ kN} > 512 \text{ kN} \quad \mathbf{O.K.}$$

*Resistance About Axis X-X and Y-Y Using the Tables of Factored Axial Compressive Resistances of Double-Angle Struts, Long Legs Back-to-Back in Part 4.*

For the axis Y-Y the calculated compressive resistance should be as follows:

$$\begin{aligned}
 \lambda &= \sqrt{\frac{F_y}{F_e}} \\
 &= \sqrt{\frac{350}{398}} \\
 &= 0.94
 \end{aligned}$$

$$\begin{aligned}
 C_r &= \frac{\phi A F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}} \\
 &= \frac{0.9 \times 3440 \text{ mm}^2 \times 350 \text{ MPa}}{(1 + 0.94^{2 \times 1.34})^{\frac{1}{1.34}}}
 \end{aligned}$$

$$= 685 \text{ kN}$$

The actual length is replaced by an equivalent length based on the radius of gyration of the built-up section  $r'_y$  and the slenderness ratio of the component angles.

$$\begin{aligned} L_e &= r_y \sqrt{\left(\frac{KL}{r'}\right)_y^2 + \left(\frac{KL}{r}\right)_z^2} \\ &= 39.7 \times \sqrt{\left(\frac{2400}{43.5}\right)^2 + \left(\frac{0.65 \times 800}{18.5}\right)^2} \\ &= 2458 \text{ mm} \end{aligned}$$

The factored axial compressive resistance is taken from the lower (Y-Y) portion of CISC Handbook Part 4 as:

$$C_r = 695 \text{ kN} > 512 \text{ kN} \quad \mathbf{O.K.}$$

The factored axial compressive resistance is taken from the lower (X-X) portion of CISC Handbook Part 4 as:

$$L = 2400 \text{ mm}$$

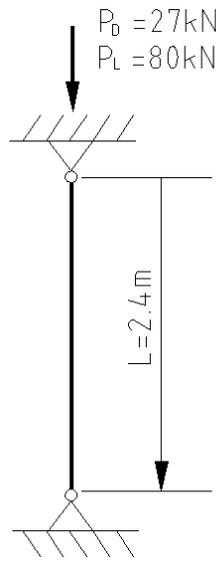
$$C_r = 649 \text{ kN} > 512 \text{ kN} \quad \mathbf{O.K.}$$

$$C_r (\mathbf{S - STEEL}) = 648 \text{ kN} (0.15 \% \text{ Diff.})$$

Therefore, X-X axis flexural buckling governs.

**EXAMPLE E.6 DOUBLE ANGLE COMPRESSION MEMBER WITH SLENDER ELEMENTS**
**Given:**

Determine if a 2L 127x76x6.4 LLBB (20mm separation) strut, CSA G40.21- 350W, with a length of 2.4m and pinned ends has sufficient resistance to support a dead load of 27kN and live load of 80kN. Also, calculate the required number of pretensioned bolted or welded intermediate connectors.


**Solution:**

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 350W

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

L 127x76x6.4 LLBB

$$t = 6.35 \text{ mm}$$

$$y = 42.1 \text{ mm}$$

$$r_z = 16.8 \text{ mm}$$

2L 127x76x6.4 LLBB

$$r_x = 41.2 \text{ mm}$$

$$r_y = 30.8 \text{ mm for 10mm separation}$$

$$r_y = 34.5 \text{ mm for 20mm separation}$$

$$A = 2500 \text{ mm}^2$$

From Division B Part 4 of NBCC 2015, the factored compressive force is:

$$C_f = 1.25D + 1.5L = 1.25 \times 27 + 1.5 \times 80 = 154 \text{ kN}$$

*Class of Section*

Width-to-thickness ratios, based on S16-14 Clause 11.2, are as follows:

$$\frac{d}{t} = \frac{127}{6.35} = 20 > \frac{250}{\sqrt{F_y}} = 13.4$$

$$\frac{b}{t} = \frac{76.2}{6.35} = 12 < \frac{250}{\sqrt{F_y}} = 13.4$$

The angle is therefore a Class 4 section in axial compression.

*Effective Area*

From Clause 13.3.5(a), the effective area shall be as follows:

$$\begin{aligned} A_e &= A - 2\left(\frac{d}{t} - \frac{250}{\sqrt{F_y}}\right)t^2 \\ &= 2500 - 2\left(20 - \frac{250}{\sqrt{350}}\right)6.35^2 \\ &= 1965 \text{ mm}^2 \end{aligned}$$

*Compressive Resistance About Axis X-X, Flexural Mode*

From Clause 13.3.1, the elastic buckling stress  $F_e$  shall be as follows:

$$\frac{KL_x}{r_x} = \frac{1.0 \times 2400}{41.2} = 58$$

$$\begin{aligned} F_e &= \frac{\pi^2 E}{\left(\frac{KL_x}{r_x}\right)^2} \\ &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(58)^2} \\ &= 587 \text{ MPa} \end{aligned}$$

$$\begin{aligned} \lambda &= \sqrt{\frac{F_y}{F_e}} \\ &= \sqrt{\frac{350}{587}} \\ &= 0.772 \end{aligned}$$

$$\begin{aligned}
 C_{rx} &= \frac{\phi A_e F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}} \\
 &= \frac{0.9 \times 1965 \text{ mm}^2 \times 350 \text{ MPa}}{(1 + 0.772^{2 \times 1.34})^{\frac{1}{1.34}}} \\
 &= 457 \text{ MPa}
 \end{aligned}$$

*Compressive Resistance About Axis Y-Y, Torsional-Flexural Mode (Detailed Calculation)*

Based on Clause 13.3.2, the shear center location is as follows:

$$x_o = 0 \quad y_o = y - \frac{t}{2} = 42.1 - \frac{6.35}{2} = 38.9 \text{ mm}$$

Torsional-flexural section properties are calculated as below.

$$\begin{aligned}
 \bar{r}_o^2 &= x_o^2 + y_o^2 + r_x^2 + r_y^2 \\
 &= 0 + (38.9 \text{ mm})^2 + (41.2 \text{ mm})^2 + (34.5 \text{ mm})^2 \\
 &= 4401 \text{ mm}^2
 \end{aligned}$$

$$\begin{aligned}
 \Omega &= 1 - \left( \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \right) \\
 &= 1 - \left( \frac{0^2 + 38.9^2}{4401} \right) \\
 &= 0.66
 \end{aligned}$$

Based on Clause 19.2.4(b), the slenderness ratio of the built-up member shall be as follows:

$$\rho_o = \frac{KL_y}{r_y} = \frac{1 \times 2400}{34.5} = 69.56$$

Equivalent slenderness ratio of a component angle, with two welded intermediate connectors spaced at  $L/3 = 2400/3 = 800 \text{ mm}$  and  $K = 0.65$ , is calculated as follows:

$$\begin{aligned}
 \rho_i &= \frac{KL_z}{r_z} = \frac{0.65 \times 800}{16.8} = 30.95 \\
 \rho_e &= \sqrt{\rho_o^2 + \rho_i^2} = \sqrt{69.56^2 + 30.95^2} = 76.13
 \end{aligned}$$

*Elastic Torsional Buckling Stress*

From Clause 13.3.2.b), the factored compressive resistance for singly symmetric sections, with the y-axis taken as the

axis of symmetry, is the lesser of  $F_{ex}$  and  $F_{eyz}$ .

$$\begin{aligned}
 F_{ey} &= \frac{\pi^2 E}{\rho_e^2} \\
 &= \frac{\pi^2 \times 200 \times 10^3}{76.13^2} \\
 &= 340.5 \text{ MPa}
 \end{aligned}$$

$$\begin{aligned}
 F_{ez} &= \left( \frac{\pi^2 E C_w}{(K L_z)^2} + GJ \right) \frac{1}{A \bar{r}_o^2} \\
 &= \left( \frac{\pi^2 (200 \times 10^3 \text{ MPa}) (2 \times 16.3 \times 10^6 \text{ mm}^6)}{(1 \times 2400 \text{ mm})^2} + (77 \times 10^3 \text{ MPa}) (2 \times 16.8 \times 10^3 \text{ mm}^4) \right) \\
 &\quad \times \frac{1}{(2500 \text{ mm}^2)(4401 \text{ mm}^2)} \\
 &= 236 \text{ MPa}
 \end{aligned}$$

$$\begin{aligned}
 F_e &= F_{eyz} = \frac{F_{ey} + F_{ez}}{2\Omega} \left( 1 - \sqrt{1 - \frac{4F_{ey}F_{ez}\Omega}{(F_{ey} + F_{ez})^2}} \right) \\
 &= \frac{340.5 + 236}{2 \times 0.66} \left( 1 - \sqrt{1 - \frac{4 \times 340.5 \times 236 \times 0.66}{(340.5 + 236)^2}} \right) \\
 &= 174 \text{ MPa} \quad \text{(governs)}
 \end{aligned}$$

*Factored Compressive Resistance*

$$\begin{aligned}
 \lambda &= \sqrt{\frac{F_y}{F_e}} \\
 &= \sqrt{\frac{350}{174}} \\
 &= 1.42
 \end{aligned}$$

$$\begin{aligned}
 C_r &= \frac{\phi A_e F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}} \\
 &= \frac{0.9 \times 1965 \text{ mm}^2 \times 350 \text{ MPa}}{(1 + 1.42^{2 \times 1.34})^{\frac{1}{1.34}}} \\
 &= 240 \text{ kN} > 224 \text{ kN} \quad \text{O.K.}
 \end{aligned}$$

$$C_r (\mathbf{S} - \mathbf{STEEL}) = 240 \text{ kN (0.00 \% Diff.)}$$

*Resistance About Axis Y-Y Using the Tables of Factored Axial Compressive Resistances of Double-Angle Struts, Long Legs Back-to-Back in Part 4.*

The actual length is replaced by an equivalent length based on the radius of gyration of the built-up section  $r'_y$  and the slenderness ratio of the component angles.

$$\begin{aligned} L_e &= r_y \sqrt{\left(\frac{KL}{r'}\right)_y^2 + \left(\frac{KL}{r}\right)_z^2} \\ &= 30.8 \times \sqrt{\left(\frac{2400}{34.5}\right)^2 + \left(\frac{0.65 \times 800}{16.8}\right)^2} \\ &= 2345 \text{ mm} \end{aligned}$$

The factored axial compressive resistance is taken from the lower (Y-Y) portion of CISC Handbook Part 4 as:

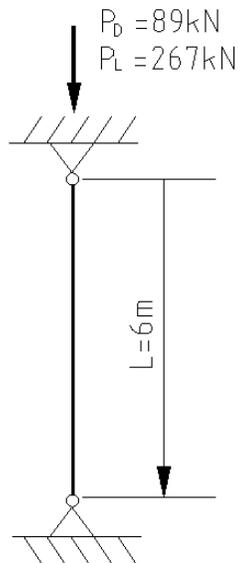
$$C_r = 244.4 \text{ kN} > 224 \text{ kN} \quad \mathbf{O.K.}$$

Therefore, Y-Y axis torsional-flexural buckling governs.

## EXAMPLE E.7 WT COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

### Given:

Select a CSA G40.21- 350W nonslender WT-shape compression member with a length of 6m to support a dead load of 89kN and 267kN in axial compression. The ends are pinned.



### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 350W

$$F_y = 350\text{ MPa}$$

$$F_u = 450\text{ MPa}$$

From Division B Part 4 of NBCC 2015, the factored compressive force is:

$$C_f = 1.25D + 1.5L = 1.25 \times 89 + 1.5 \times 267 = 512\text{ kN}$$

#### Calculation Solution

From CISC *Handbook* Annex F Clause F.2, for a pinned-pinned condition,  $K = 1.0$ .

Therefore,  $(KL)_x = (KL)_y = 6\text{ m}$ .

From CISC *Handbook* Part 6, the geometric properties are as follows:

Try a WT180x50.5

$$A_g = 6450 \text{ mm}^2$$

$$r_x = 46.1 \text{ mm}$$

$$r_y = 62.6 \text{ mm}$$

$$C_w = 863 \times 10^6 \text{ mm}^6$$

$$J = 626 \times 10^3 \text{ mm}^4$$

$$y = 32.7 \text{ mm}$$

$$I_x = 13.7 \times 10^6 \text{ mm}^4$$

$$I_y = 25.3 \times 10^6 \text{ mm}^4$$

$$d = 179 \text{ mm}$$

$$w = 10.5 \text{ mm}$$

$$b = 255 \text{ mm}$$

$$t = 18.3 \text{ mm}$$

### Slenderness Check

Based on S16-14 Table 1, the maximum width-to-thickness ratios are checked as follows:

$$\frac{b_{el}}{t} = \frac{d}{w} = \frac{179}{10.5} = 17.05 < \frac{340}{\sqrt{F_y}} = 18.17$$

Therefore, the stem is not slender.

$$\frac{b_{el}}{t} = \frac{b/2}{t} = \frac{255/2}{18.3} = 6.97 < \frac{200}{\sqrt{F_y}} = 10.69$$

Therefore, the flange is not slender.

### Compressive Resistance About Axis X-X, Flexural Mode

From Clause 13.3.1, the elastic buckling stress  $F_e$  shall be as follows:

$$\frac{KL_x}{r_x} = \frac{1.0 \times 6000}{46.1} = 130$$

$$\begin{aligned} F_{ex} &= \frac{\pi^2 E}{\left(\frac{KL_x}{r_x}\right)^2} \\ &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(130)^2} \\ &= 116.8 \text{ MPa} \quad (\text{governs}) \end{aligned}$$

$$\begin{aligned}\lambda &= \sqrt{\frac{F_y}{F_e}} \\ &= \sqrt{\frac{350}{116.8}} \\ &= 1.73\end{aligned}$$

$$\begin{aligned}C_{rx} &= \frac{\phi A_g F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}} \\ &= \frac{0.9 \times 6450 \text{ mm}^2 \times 350 \text{ MPa}}{(1 + 1.73^{2 \times 1.34})^{\frac{1}{1.34}}} \\ &= 581.6 \text{ MPa}\end{aligned}$$

*Compressive Resistance About Axis Y-Y, Torsional-Flexural Mode*

$$\frac{KL_y}{r_y} = \frac{1.0 \times 6000}{62.6} = 96$$

$$\begin{aligned}F_{ey} &= \frac{\pi^2 E}{\left(\frac{KL_y}{r_y}\right)^2} \\ &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(96)^2} \\ &= 214 \text{ MPa}\end{aligned}$$

Based on Clause 13.3.2, the shear center location is as follows:

$$x_o = 0 \quad y_o = y - \frac{t}{2} = 32.7 - \frac{18.3}{2} = 23.6 \text{ mm}$$

Torsional-flexural section properties are calculated as below.

$$\begin{aligned}\bar{r}_o^2 &= x_o^2 + y_o^2 + r_x^2 + r_y^2 \\ &= 0 + (23.6 \text{ mm})^2 + (46.1 \text{ mm})^2 + (62.6 \text{ mm})^2 \\ &= 6601 \text{ mm}^2\end{aligned}$$

$$\Omega = 1 - \left( \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \right)$$

$$= 1 - \left( \frac{0^2 + 23.6^2}{6601} \right)$$

$$= 0.92$$

### Elastic Torsional Buckling Stress

From Clause 13.3.2.b), the factored compressive resistance for singly symmetric sections, with the y-axis taken as the axis of symmetry, is the lesser of  $F_{ex}$  and  $F_{eyz}$ .

$$F_{ez} = \left( \frac{\pi^2 EC_w}{(KL_z)^2} + GJ \right) \frac{1}{Ar_o^2}$$

$$= \left( \frac{\pi^2 (200 \times 10^3 \text{ MPa}) (863 \times 10^6 \text{ mm}^6)}{(1 \times 6000 \text{ mm})^2} + (77 \times 10^3 \text{ MPa}) (626 \times 10^3 \text{ mm}^4) \right)$$

$$\times \frac{1}{(6450 \text{ mm}^2)(6601 \text{ mm}^2)}$$

$$= 1133 \text{ MPa}$$

$$F_e = F_{eyz} = \frac{F_{ey} + F_{ez}}{2\Omega} \left( 1 - \sqrt{1 - \frac{4F_{ey}F_{ez}\Omega}{(F_{ey} + F_{ez})^2}} \right)$$

$$= \frac{214 + 1133}{2 \times 0.92} \left( 1 - \sqrt{1 - \frac{4 \times 214 \times 1133 \times 0.92}{(214 + 1133)^2}} \right)$$

$$= 210 \text{ MPa}$$

By comparison, the flexural mode of buckling about axis X-X is governing,  $F_e = 116.8 \text{ MPa}$ .

### Factored Compressive Resistance

$$C_r = C_{rx} = 581.6 \text{ MPa} > 512 \text{ kN} \quad \text{O.K.}$$

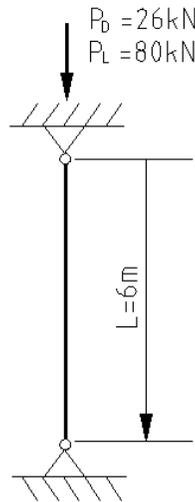
$$C_r (\text{S - STEEL}) = 580 \text{ kN} (0.28\% \text{ Diff.})$$

Therefore, X-X axis flexural buckling governs.

## EXAMPLE E.8 WT COMPRESSION MEMBER WITH SLENDER ELEMENTS

### Given:

Select a CSA G40.21- 350W WT-shape compression member with a length of 6m to support a dead load of 26kN and 80kN in axial compression. The ends are pinned.



### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 350W

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From Division B Part 4 of NBCC 2015, the factored compressive force is:

$$C_f = 1.25D + 1.5L = 1.25 \times 26 + 1.5 \times 80 = 153 \text{ kN}$$

#### Calculation Solution

From CISC *Handbook* Annex F Clause F.2, for a pinned-pinned condition,  $K = 1.0$ .

Therefore,  $(KL)_x = (KL)_y = 6 \text{ m}$ .

From CISC *Handbook* Part 6, the geometric properties are as follows:

Try a WT180x22.5

$$A_g = 2860 \text{ mm}^2$$

$$\begin{aligned}
 r_x &= 52.7 \text{ mm} \\
 r_y &= 37.8 \text{ mm} \\
 C_w &= 78.4 \times 10^6 \text{ mm}^6 \\
 J &= 79.4 \times 10^3 \text{ mm}^4 \\
 y &= 40.2 \text{ mm} \\
 I_x &= 7.96 \times 10^6 \text{ mm}^4 \\
 I_y &= 4.09 \times 10^6 \text{ mm}^4 \\
 d &= 176 \text{ mm} \\
 w &= 6.9 \text{ mm} \\
 b &= 171 \text{ mm} \\
 t &= 9.8 \text{ mm}
 \end{aligned}$$

### Slenderness Check

Based on S16-14 Table 1, the maximum width-to-thickness ratios are checked as follows:

$$\frac{b_{el}}{t} = \frac{d}{w} = \frac{176}{6.9} = 25.5 > \frac{340}{\sqrt{F_y}} = 18.17$$

Therefore, the stem is slender.

$$\frac{b_{el}}{t} = \frac{b/2}{t} = \frac{171/2}{9.8} = 8.7 < \frac{200}{\sqrt{F_y}} = 10.69$$

Therefore, the flange is not slender.

### Effective Area

From Clause 13.3.5(a), the effective area shall be as follows:

$$\begin{aligned}
 A_e &= A - \left( \frac{d}{w} - \frac{340}{\sqrt{F_y}} \right) w^2 \\
 &= 2860 - \left( 25.5 - \frac{340}{\sqrt{350}} \right) 6.9^2 \\
 &= 2511 \text{ mm}^2
 \end{aligned}$$

### Compressive Resistance About Axis X-X, Flexural Mode

From Clause 13.3.1, the elastic buckling stress  $F_e$  shall be as follows:

$$\frac{KL_x}{r_x} = \frac{1.0 \times 6000}{52.7} = 114$$

$$\begin{aligned}
 F_{ex} &= \frac{\pi^2 E}{\left(\frac{KL_x}{r_x}\right)^2} \\
 &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(114)^2} \\
 &= 152 \text{ MPa}
 \end{aligned}$$

$$\begin{aligned}
 \lambda &= \sqrt{\frac{F_y}{F_e}} \\
 &= \sqrt{\frac{350}{152}} \\
 &= 1.52
 \end{aligned}$$

$$\begin{aligned}
 C_{rx} &= \frac{\phi A_e F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}} \\
 &= \frac{0.9 \times 2511 \text{ mm}^2 \times 350 \text{ MPa}}{(1 + 1.52^{2 \times 1.34})^{\frac{1}{1.34}}} \\
 &= 277 \text{ MPa}
 \end{aligned}$$

*Compressive Resistance About Axis Y-Y, Torsional-Flexural Mode*

$$\frac{KL_y}{r_y} = \frac{1.0 \times 6000}{37.8} = 158.7$$

$$\begin{aligned}
 F_{ey} &= \frac{\pi^2 E}{\left(\frac{KL_y}{r_y}\right)^2} \\
 &= \frac{\pi^2 (200 \times 10^3 \text{ MPa})}{(158.7)^2} \\
 &= 78.4 \text{ MPa}
 \end{aligned}$$

Based on Clause 13.3.2, the shear center location is as follows:

$$x_o = 0 \quad y_o = y - \frac{t}{2} = 40.2 - \frac{9.8}{2} = 35.3 \text{ mm}$$

Torsional-flexural section properties are calculated as below.

$$\bar{r}_o^2 = x_o^2 + y_o^2 + r_x^2 + r_y^2$$

$$\begin{aligned}
 &= 0 + (35.3\text{mm})^2 + (52.7\text{mm})^2 + (37.8\text{mm})^2 \\
 &= 5452 \text{ mm}^2
 \end{aligned}$$

$$\begin{aligned}
 \Omega &= 1 - \left( \frac{x_o^2 + y_o^2}{\bar{r}_o^2} \right) \\
 &= 1 - \left( \frac{0^2 + 35.3^2}{5452} \right) \\
 &= 0.77
 \end{aligned}$$

### Elastic Torsional Buckling Stress

From Clause 13.3.2.b), the factored compressive resistance for singly symmetric sections, with the y-axis taken as the axis of symmetry, is the lesser of  $F_{ex}$  and  $F_{eyz}$ .

$$\begin{aligned}
 F_{ez} &= \left( \frac{\pi^2 EC_w}{(KL_z)^2} + GJ \right) \frac{1}{A\bar{r}_o^2} \\
 &= \left( \frac{\pi^2 (200 \times 10^3 \text{ MPa}) (78.4 \times 10^6 \text{ mm}^6)}{(1 \times 6000 \text{ mm})^2} + (77 \times 10^3 \text{ MPa}) (79.4 \times 10^3 \text{ mm}^4) \right) \\
 &\quad \times \frac{1}{(2860 \text{ mm}^2)(5452 \text{ mm}^2)} \\
 &= 392 \text{ MPa}
 \end{aligned}$$

$$\begin{aligned}
 F_e &= F_{eyz} = \frac{F_{ey} + F_{ez}}{2\Omega} \left( 1 - \sqrt{1 - \frac{4F_{ey}F_{ez}\Omega}{(F_{ey} + F_{ez})^2}} \right) \\
 &= \frac{78.4 + 392}{2 \times 0.77} \left( 1 - \sqrt{1 - \frac{4 \times 78.4 \times 392 \times 0.77}{(78.4 + 392)^2}} \right) \\
 &= 74.4 \text{ MPa} \quad (\text{governs})
 \end{aligned}$$

By comparison, the torsional-flexural mode of buckling about axis Y-Y is governing,  $F_e = 74.4 \text{ MPa}$ .

### Factored Compressive Resistance

$$\begin{aligned}
 \lambda &= \sqrt{\frac{F_y}{F_e}} \\
 &= \sqrt{\frac{350}{74.4}} \\
 &= 2.17
 \end{aligned}$$

$$C_r = \frac{\phi A_e F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}}$$
$$= \frac{0.9 \times 2511 \text{mm}^2 \times 350 \text{MPa}}{(1 + 2.17^{2 \times 1.34})^{\frac{1}{1.34}}}$$
$$= 154 \text{ kN} > 153 \text{ kN} \quad \mathbf{O.K.}$$

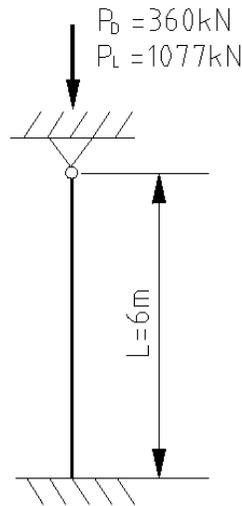
$$C_r (\mathbf{S - STEEL}) = 154 \text{ kN} (0.00\% \text{ Diff.})$$

Therefore, Y-Y axis torsional-flexural buckling governs.

### EXAMPLE E.9 RECTANGULAR HSS COMPRESSION MEMBER WITHOUT SLENDER ELEMENTS

#### Given:

Select a CSA G40.21- 350W Class C rectangular HSS compression member, with a length of 6 m, to support a dead load of 360kN and 1077kN in axial compression. The base is fixed and the top is pinned.



#### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 350W Class C

$$F_y = 350\text{ MPa}$$

$$F_u = 450\text{ MPa}$$

From Division B Part 4 of NBCC 2015, the factored compressive force is:

$$C_f = 1.25D + 1.5L = 1.25 \times 360 + 1.5 \times 1077 = 2066\text{ kN}$$

#### Table Solution

From CISC *Handbook* Annex F Clause F.2, for one end fixed and one end pinned condition,  $K = 0.8$ .

$$\text{Therefore, } (KL)_x = (KL)_y = 0.8 \times 6\text{ m} = 4.8\text{ m}$$

Search the table with an effective length,  $KL = 4800\text{ mm}$ , and proceed across the table in CISC *Handbook* Part 4 until reaching the least weight shape with a factored compressive resistance that equals or exceeds the factored axial compressive force. Select a HSS305x203x9.5.

From the table in CISC *Handbook* Part 4, the factored compressive resistance for the effective length of 4800 mm is:

$$C_r = 2116 \text{ kN} > 2066 \text{ kN} \quad \text{O.K.}$$

The factored compressive resistance can be verified by hand calculations as follows.

#### *Calculation Solution*

From CISC *Handbook* Part 6, the geometric properties are as follows:

HSS305x203x9.5

$$A_g = 9090 \text{ mm}^2$$

$$r_x = 113 \text{ mm}$$

$$r_y = 82.7 \text{ mm}$$

$$b = 203.2 \text{ mm}$$

$$d = 304.8 \text{ mm}$$

$$t = 9.53 \text{ mm}$$

#### *Slenderness Check*

Based on S16-14 Table 1, the maximum width-to-thickness ratios shall be checked. Calculate b/t of the slenderest wall.

Note: According to Clause 11.3.2(b), the nominal outside dimension minus four times the wall thickness.

$$\frac{b_{el}}{t} = \frac{d - 4t}{t} = \frac{304.8 - 4 \times 9.53}{9.53} = 27.98 < \frac{670}{\sqrt{F_y}} = 35.81$$

Therefore, the section does not contain slender elements.

Because  $r_y < r_x$  and  $(KL)_x = (KL)_y$ ,  $r_y$  will govern the factored compressive resistance.

#### *Compressive Resistance About Axis Y-Y, Flexural Mode*

From Clause 13.3.1, the elastic buckling stress  $F_e$  shall be as follows:

$$\frac{KL_y}{r_y} = \frac{0.8 \times 6000}{82.7} = 58$$

$$F_{ey} = \frac{\pi^2 E}{\left(\frac{KL_y}{r_y}\right)^2}$$

$$\begin{aligned}
 &= \frac{\pi^2(200 \times 10^3 \text{MPa})}{(58)^2} \\
 &= 587 \text{ MPa} \quad (\text{governs})
 \end{aligned}$$

$$\begin{aligned}
 \lambda &= \sqrt{\frac{F_y}{F_e}} \\
 &= \sqrt{\frac{350}{587}} \\
 &= 0.772
 \end{aligned}$$

$$\begin{aligned}
 C_{ry} &= \frac{\phi A_g F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}} \\
 &= \frac{0.9 \times 9090 \text{mm}^2 \times 350 \text{MPa}}{(1 + 0.772^{2 \times 1.34})^{\frac{1}{1.34}}} \\
 &= 2116 \text{ kN}
 \end{aligned}$$

*Factored Compressive Resistance*

$$C_r = C_{ry} = 2116 \text{ kN} > 2066 \text{ kN} \quad \text{O.K.}$$

$$C_r (\text{S} - \text{STEEL}) = 2114 \text{ kN} (0.09\% \text{ Diff.})$$

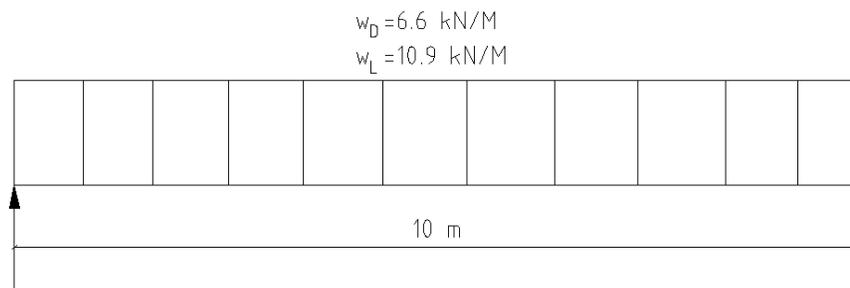
Therefore, Y-Y axis flexural buckling governs.

## Chapter F Design of Members for Flexure

### EXAMPLE F.1-1A W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, CONTINUOUSLY BRACED

#### Given:

Select a W-shape ASTM A992 beam with a simple span of 10 m. Limit the member to a maximum nominal depth of 460mm. Limit the live load deflection to  $L/360$ . The specified loads are a uniform dead load of 6.6 kN/m and a uniform live load of 10.9 kN/m. Assume the beam is continuously braced.



Beam Loading & Bracing Diagram  
(full lateral support)

#### Solution:

From CISC *Handbook* Part 6, the material properties are as follows:

ASTM A992

$$F_y = 345 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From Division B Part 4 of NBCC 2015, the bending moment under factored load is:

$$w = 1.25D + 1.5L = 1.25 \times 6.6 + 1.5 \times 10.9 = 24.6 \text{ kN/m}$$

$$M_f = \frac{24.6 \text{ kN/m} \times (10 \text{ m})^2}{8} = 307.5 \text{ kN-m}$$

Required Moment of Inertia for Live-Load Deflection Criterion of  $L/360$

$$\Delta_{max} = \frac{L}{360}$$

$$= \frac{10 \text{ m} \left( \frac{1000 \text{ mm}}{\text{m}} \right)}{360}$$

$$= 27.8\text{mm}$$

$$I_{reqd} = \frac{5w_L l^4}{384E\Delta_{max}} \text{ from CISC Handbook Part 5}$$

$$= \frac{5 \times 10.9\text{kN/m} \times (10 \times 1000)^4}{384 \times (200 \times 10^3\text{MPa}) \times 27.8\text{mm}}$$

$$= 255 \times 10^6 \text{ mm}^4$$

### Beam Selection

Select a W460x74 from table in CISC Handbook Part 5.

$$M_r = 512 \text{ kN} - m > 307.5 \text{ kN} - m \quad \mathbf{O.K.}$$

$$I_x = 332 \times 10^6 \text{ mm}^4 > 255 \times 10^6 \text{ mm}^4 \quad \mathbf{O.K.}$$

$$M_r (\mathbf{S - STEEL}) = 512\text{kN} - m \text{ (0.0\% Diff.)}$$

## EXAMPLE F.1-1B W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, CONTINUOUSLY BRACED

### Given:

Verify the factored moment resistance of the W460x74, ASTM A992 beam selected in Example F.1-1A by applying the requirements of the CSA S16-14 directly.

### Solution:

From CISC *Handbook* Part 6, the material properties are as follows:

$$\begin{aligned} &W460x74 \\ &ASTM A992 \\ &F_y = 345 \text{ MPa} \\ &F_u = 450 \text{ MPa} \end{aligned}$$

From CISC *Handbook* Part 6, the material properties are as follows:

$$\begin{aligned} &W460x74 \\ &Z_x = 1650 \times 10^3 \text{ mm}^3 \end{aligned}$$

The required moment resistance from Example F.1-1A is,  $M_f = 307.5 \text{ kN} - \text{m}$ .

*Factored Moment Resistance,  $M_r$*

According to CISC *Handbook* Table 5-1, the section is Class 1. Because the beam is continuously braced and the section is Class 1, all fibres of the section are completely yielded.

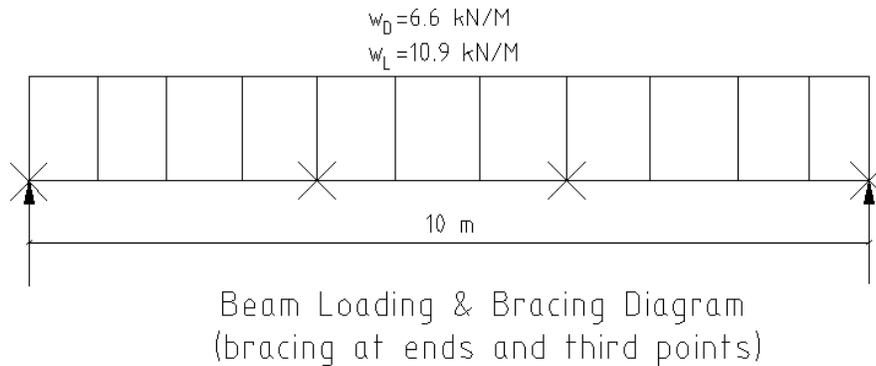
$$\begin{aligned} M_r &= \phi Z_x F_y \\ &= 0.9 \times 1650 \times 10^3 \text{ mm}^3 \times 345 \text{ MPa} \\ &= 512 > 307.5 \text{ kN} - \text{m} \quad \quad \quad \mathbf{O.K} \end{aligned}$$

$$M_r (\mathbf{S} - \mathbf{STEEL}) = 512 \text{ kN} - \text{m} (0.0\% \text{ Diff.})$$

### EXAMPLE F.1-2A W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT THIRD POINTS

#### Given:

Verify the factored bending moment of the W460x74, ASTM A992 beam selected in Example F.1-1A with the beam braced at the ends and third points. Use the CISC Handbook.



#### Solution:

From CISC *Handbook* Part 6, the material properties are as follows:

$$\begin{aligned} & \text{ASTM A992} \\ & F_y = 345 \text{ MPa} \\ & F_u = 450 \text{ MPa} \end{aligned}$$

The required bending moment at midspan from Example F.1-1A is  $\square$

$$M_f = \frac{24.6 \text{ kN/m} \times (10 \text{ m})^2}{8} = 307.5 \text{ kN} \cdot \text{m}$$

*Unbraced Length*

$$\begin{aligned} L_b &= \frac{10 \text{ m} \times 1000 (\text{mm/m})}{3} \\ &= 3333 \text{ mm} \end{aligned}$$

*Required Moment of Inertia for Live-Load Deflection Criterion of L/360*

$$W = 10.9 \text{ kN/m} \times 10 \text{ m} = 109 \text{ kN}$$

From CISC *Handbook* Table 5-8,  $B_d = 1.0$ .

Using the graph from CISC *Handbook* Figure 5-2:

$$C_d = 2.35 \times 10^6 \text{ mm}^4 / \text{kN} \quad (\text{for } \frac{L}{\Delta} = 360 \text{ and } L = 10\text{m})$$

$$I_{reqd} = WC_d B_d = 109 \text{ kN} \times 2.35 \times 10^6 \text{ mm}^4 / \text{kN} \times 1.0 = 256.15 \times 10^6 \text{ mm}^4$$

Factored Moment Resistance,  $M_r$

From CISC *Handbook* Part 5, for the W460X74 section, the factored moment resistance with the unbraced length of 3333mm is as follows:

$$M_r = 461 \text{ kN} - \text{m} > 307.5 \text{ kN} - \text{m} \quad \mathbf{O.K.}$$

$$I_x = 332 \times 10^6 \text{ mm}^4 > 256.15 \times 10^6 \text{ mm}^4 \quad \mathbf{O.K.}$$

$$M_r (\mathbf{S} - \mathbf{STEEL}) = 479 \text{ kN} - \text{m} \text{ (3.9\% Diff.)}$$

## EXAMPLE F.1-2B W-SHAPE FLEXURAL MEMBER DESIGN IN STRONG-AXIS BENDING, BRACED AT THIRD POINTS

### Given:

Verify the factored moment resistance of the W460x74, ASTM A992 beam selected in Example F.1-1A by applying the requirements of the CSA S16-14 directly.

### Solution:

From CISC *Handbook* Part 6, the material properties are as follows:

$$\begin{aligned} & \text{W460x74} \\ & \text{ASTM A992} \\ & F_y = 345 \text{ MPa} \\ & F_u = 450 \text{ MPa} \end{aligned}$$

From CISC *Handbook* Part 6, the material properties are as follows:

$$\begin{aligned} & \text{W460x74} \\ & Z_x = 1650 \times 10^3 \text{ mm}^3 \\ & I_y = 16.6 \times 10^6 \text{ mm}^4 \\ & C_w = 813 \times 10^9 \text{ mm}^6 \\ & J = 516 \times 10^3 \text{ mm}^4 \end{aligned}$$

The required moment resistance from Example F.1-1A is,  $M_f = 307.5 \text{ kN} - \text{m}$ .

*Factored Moment Resistance,  $M_r$*

According to Clause 13.6(a), the factored moment resistance,  $M_r$ , of a segment between effective brace points shall be determined as follows:

$$\omega_2 = \frac{4M_{max}}{\sqrt{M_{max}^2 + 4M_a^2 + 7M_b^2 + 4M_c^2}} \leq 2.5$$

For the center segment of the beam, the required moments for the above equation can be calculated as a percentage of the maximum midspan moment as:  $M_{max} = 1.00$ ,  $M_a = 0.972$ ,  $M_b = 1.00$  and  $M_c = 0.972$ .

$$\omega_2 = \frac{4 \times 1.00}{\sqrt{(1.00)^2 + 4 \times (0.972)^2 + 7 \times (1.00)^2 + 4 \times (0.972)^2}} = 1.01$$

For the end-span segment of the beam, the required moments for the above equation can be calculated as a percentage

of the maximum midspan moment as:  $M_{max} = 0.889$ ,  $M_a = 0.306$ ,  
 $M_b = 0.556$  and  $M_c = 0.750$ .

$$\omega_2 = \frac{4 \times 0.889}{\sqrt{(0.889)^2 + 4 \times (0.306)^2 + 7 \times (0.556)^2 + 4 \times (0.750)^2}} = 1.51$$

Thus, the center span with the higher required moment resistance and lower  $\omega_2$ , will govern.

$$M_u = \frac{\omega_2 \pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w}$$

$$= \frac{1.01 \times \pi}{3333 \text{ mm}} \times \sqrt{(200 \times 10^3 \text{ MPa}) \times 16.6 \times 10^6 \text{ mm}^4 \times (77 \times 10^3 \text{ MPa}) \times (516 \times 10^3 \text{ mm}^4)}$$

$$\sqrt{\left(\frac{\pi \times (200 \times 10^3 \text{ MPa})}{3333 \text{ mm}}\right)^2 \times (16.6 \times 10^6 \text{ mm}^4) \times (813 \times 10^9 \text{ mm}^6)}$$

$$= 744.5 \text{ kN} - m$$

$$M_p = Z_x F_y = 1650 \times 10^3 \text{ mm}^3 \times 350 \text{ MPa} = 577.5 \text{ kN} - m$$

Since  $M_u = 744.5 \text{ kN} - m > 0.67 M_p = 0.67 \times 577.5 \text{ kN} - m = 386.9 \text{ kN} - m$  □ Clause 13.6(a)(i) shall apply.

$$M_r = 1.15 \phi M_p \left[1 - \frac{0.28 M_p}{M_u}\right]$$

$$= 1.15 \times 0.9 \times 577.5 \text{ kN} - m \times \left[1 - \frac{0.28 \times 577.5 \text{ kN} - m}{744.5 \text{ kN} - m}\right]$$

$$= 467.9 \text{ kN} - m < \phi M_p = 0.9 \times 577.5 \text{ kN} - m = 519.75 \text{ kN} - m$$

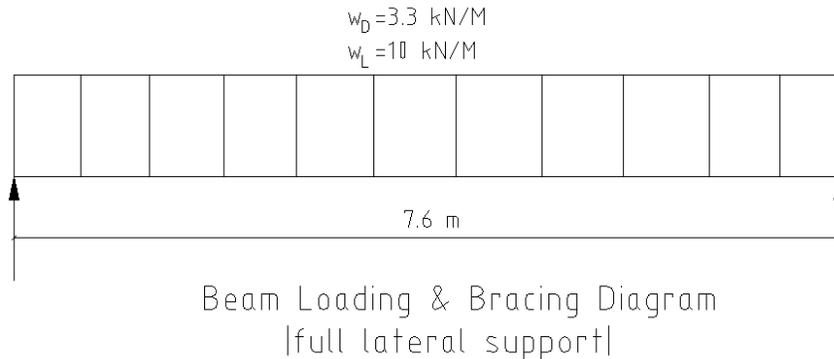
$$M_r = 467.9 \text{ kN} - m > M_f = 307.5 \text{ kN} - m \quad \mathbf{O.K.}$$

$$M_r (\mathbf{S - STEEL}) = 479 \text{ kN} - m \text{ (2.37\% Diff.)}$$

### EXAMPLE F.2-1A CLASS 3 CHANNEL FLEXURAL MEMBER, CONTINUOUSLY BRACED

#### Given:

Select a CSA G40.21- 350W channel to serve as a roof edge beam with a simple span of 7.6m. Limit the live load deflection to  $L/360$ . The specified loads are a uniform dead load of 3.3kN/m and a uniform live load of 10kN/m. The beam is continuously braced.



#### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 350W

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From Division B Part 4 of NBCC 2015, the bending moment under factored load is:

$$w = 1.25D + 1.5L = 1.25 \times 3.3 + 1.5 \times 10 = 19.2 \text{ kN/m}$$

$$M_f = \frac{19.2 \text{ kN/m} \times (7.6 \text{ m})^2}{8} = 138.6 \text{ kN} - \text{m}$$

#### Beam Selection

Per the CISC *Handbook* Part 5, all CSA G40.21- 350W channels are class 3. Because the beam is class 3 and continuously braced, the full cross-sectional strength ( $M_r = \phi M_y$ ) can be attained. Try C380x50.

$$M_r = 216 \text{ kN} - \text{m} > 138.6 \text{ kN} - \text{m} \quad \mathbf{O.K.}$$

#### Live Load Deflection

Assume the live load deflection at the center of the beam is limited to  $L/360$ .

$$\begin{aligned}\Delta_{max} &= \frac{L}{360} \\ &= \frac{7.6m \left( \frac{1000mm}{m} \right)}{360} \\ &= 21.1 \text{ mm}\end{aligned}$$

For C380x50,  $I_x = 131 \times 10^6 \text{ mm}^4$  from CISC *Handbook* Part 5.

The maximum calculated deflection is:

$$\begin{aligned}\Delta_{max} &= \frac{5w_L l^4}{384EI} \text{ from CISC } Handbook \text{ Part 5} \\ &= \frac{5 \times 10kN/m \times (7.6 \times 1000)^4}{384 \times (200 \times 10^3 MPa) \times 131 \times 10^6 \text{ mm}^4} \\ &= 16.6 \text{ mm} < 21.1 \text{ mm} \quad \mathbf{O.K.}\end{aligned}$$

$$M_r (\mathbf{S - STEEL}) = 216 \text{ kN} - m \text{ (0.00\% Diff.)}$$

## EXAMPLE F.2-1B CLASS 3 CHANNEL FLEXURAL MEMBER, CONTINUOUSLY BRACED

### Given:

Example F.2-1A can be easily solved by utilizing the tables of the *CISC Handbook*. Verify the results by applying the requirements of the CSA S16-14 directly.

### Solution:

From *CISC Handbook* Part 6, the material properties are as follows:

CSA G40.21- 350W

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From *CISC Handbook* Part 6, the material properties are as follows:

C380x50

$$S_x = 687 \times 10^3 \text{ mm}^3$$

The required moment resistance from Example F.2-1A is,  $M_f = 138.6 \text{ kN} - \text{m}$ .

*Factored Moment Resistance,  $M_r$*

According to *CISC Handbook* Part 5, the section is Class 3. Because the beam is continuously braced and the section is Class 3, the full cross-sectional strength ( $M_r = \phi M_y$ ) can be attained.

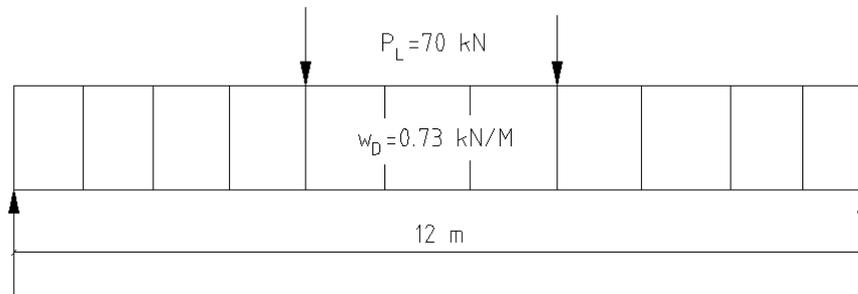
$$\begin{aligned} M_r &= \phi S_x F_y \\ &= 0.9 \times 687 \times 10^3 \text{ mm}^3 \times 350 \text{ MPa} \\ &= 216 > 138.6 \text{ kN} - \text{m} \quad \quad \quad \mathbf{O.K} \end{aligned}$$

$$M_r (\mathbf{S} - \mathbf{STEEL}) = 216 \text{ kN} - \text{m} \text{ (0.00\% Diff.)}$$

### EXAMPLE F.3A W-SHAPE FLEXURAL MEMBER WITH CLASS 3 FLANGES IN STRONG-AXIS BENDING

#### Given:

Select an ASTM A992 W-shape beam with a simple span of 12m. The specified loads are a uniform dead load of 0.73kN/m and two equal 70kN concentrated live loads acting at the third points of the beam. The beam is continuously braced. Also calculate the deflection.



Beam Loading & Bracing Diagram  
(continuous bracing)

Note: A beam with class 3 flanges will be selected to demonstrate that the tabulated values of the *CISC Handbook* account for flange slenderness.

#### Solution:

From *CISC Handbook* Part 6, the material properties are as follows:

ASTM A992

$$F_y = 345 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From Division B Part 4 of NBCC 2015, the bending moment under factored load at midspan is:

$$w = 1.25D = 1.25 \times 0.73 = 0.91 \text{ kN/m}$$

$$P = 1.5L = 1.5 \times 70 = 105 \text{ kN}$$

$$M_f = \frac{0.91 \text{ kN/m} \times (12 \text{ m})^2}{8} + 105 \times \frac{12 \text{ m}}{3} = 436 \text{ kN} \cdot \text{m}$$

#### Beam Selection

Select the lightest section providing the required bending resistance from CISC *Handbook* Part 5.

Try a W530x72. The beam has a class 3 flange.

From CISC *Handbook* Part 5, the factored bending resistance is:

$$M_r = 475 \text{ kN} - m > 436 \text{ kN} - m \quad \mathbf{O.K.}$$

$$M_r (\mathbf{S} - \mathbf{STEEL}) = 475 \text{ kN} - m \text{ (0.00\% Diff.)}$$

### Deflection

For W530x72,  $I_x = 401 \times 10^6 \text{ mm}^4$  from CISC *Handbook* Part 5.

The maximum deflection occurs at the center of the beam.

$$\begin{aligned} \Delta_{max} &= \frac{5w_D l^4}{384EI} + \frac{P_L l^3}{28EI} \text{ from CISC } Handbook \text{ Part 5} \\ &= \frac{5 \times 0.73 \text{ kN/m} \times (12 \times 1000)^4}{384 \times (200 \times 10^3 \text{ MPa}) \times 401 \times 10^6 \text{ mm}^4} + \frac{(70 \times 10^3 \text{ N}) \times (12 \times 1000)^3}{28 \times (200 \times 10^3 \text{ MPa}) \times 401 \times 10^6 \text{ mm}^4} \\ &= 56.32 \text{ mm} \end{aligned}$$

This deflection can be compared with the appropriate deflection limit for the application. Deflection will often be more critical than resistance in beam design.

### EXAMPLE F.3B W-SHAPE FLEXURAL MEMBER WITH CLASS 3 FLANGES IN STRONG-AXIS BENDING

#### Given:

Verify the results from Example F.3A by calculation using the clause from the CSA S16-14.

#### Solution:

From CISC *Handbook* Part 6, the material properties are as follows:

ASTM A992

$$F_y = 345 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

W530x72

$$t = 10.9 \text{ mm}$$

$$b = 207 \text{ mm}$$

$$d = 524 \text{ mm}$$

$$w = 9.0 \text{ mm}$$

$$S_x = 1530 \times 10^3 \text{ mm}^3$$

#### Flange Slenderness Ratio

Based on S16-14 Table 2, the maximum width-to-thickness ratio is checked as follows:

$$\frac{b_{el}}{t} = \frac{b/2}{t} = \frac{207/2}{10.9} = 9.49 < \frac{200}{\sqrt{F_y}} = 10.77 \text{ (class 3 limit)}$$

$$> \frac{170}{\sqrt{F_y}} = 9.15 \text{ (class 2 limit)}$$

$$\frac{h}{w} = \frac{d - 2t}{w} = \frac{524 - 2 \times 10.9}{9.0} = 55.8 < \frac{1100}{\sqrt{F_y}} = 59.2 \text{ (class 1 limit)}$$

Therefore, this is a class 3 section.

#### Factored Moment Resistance, $M_r$

Because the beam is continuously braced, and therefore not subject to lateral-torsional buckling, the factored resistance is governed by yielding. Clause 13.5 shall be applied to this condition.

$$M_r = \phi S_x F_y = 0.9 \times 1530 \times 10^3 \text{ mm}^3 \times 345 \text{ MPa} = 475 \text{ kN} - \text{m}$$

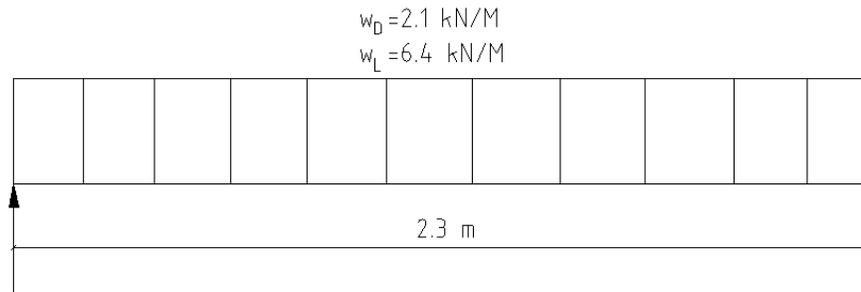
$$M_r (\text{S} - \text{STEEL}) = 475 \text{ kN} - \text{m} \text{ (0.00\% Diff.)}$$

Note that the factored resistance is identical to the tabulated value in CISC *Handbook* Part 5.

## EXAMPLE F.6 HSS FLEXURAL MEMBER WITH CLASS 1 FLANGES

### Given:

Select a square CSA G40.21- 350W Class C HSS beam to span 2.3 m. The loads are a uniform dead load of 2.1 kN/m and a uniform live load of 6.4 kN/m. Limit the live load deflection to  $L/240$ . The beam is continuously braced.



Beam Loading & Bracing Diagram  
(full lateral support)

### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 350W Class C

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From Division B Part 4 of NBCC 2015, the bending moment under factored load is:

$$w = 1.25D + 1.5L = 1.25 \times 2.1 + 1.5 \times 6.4 = 12.2 \text{ kN/m}$$

$$M_f = \frac{12.2 \text{ kN/m} \times (2.3 \text{ m})^2}{8} = 8 \text{ kN} \cdot \text{m}$$

### Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\begin{aligned} \Delta_{max} &= \frac{L}{240} \\ &= \frac{2.3 \text{ m} \left( \frac{1000 \text{ mm}}{\text{m}} \right)}{240} \\ &= 9.58 \text{ mm} \end{aligned}$$

Determine the minimum required  $I$  as follows.

$$\begin{aligned}
 I_{reqd} &= \frac{5w_L l^4}{384E\Delta_{max}} \text{ from CISC Handbook Part 5} \\
 &= \frac{5 \times 6.4 \text{ kN/m} \times (2.3 \times 1000)^4}{384 \times (200 \times 10^3 \text{ MPa}) \times 9.58 \text{ mm}} \\
 &= 1.22 \times 10^6 \text{ mm}^4
 \end{aligned}$$

### Beam Selection

Select an HSS with a minimum  $I_x$  of  $1.22 \times 10^6 \text{ mm}^4$ , using CISC Handbook Part 6, and having adequate factored resistance, using CISC Handbook Part 4.

Try an HSS76x76x6.4.

From CISC Handbook Part 6,  $I_x = 1.31 \times 10^6 \text{ mm}^4 > 1.22 \times 10^6 \text{ mm}^4$       **O.K.**

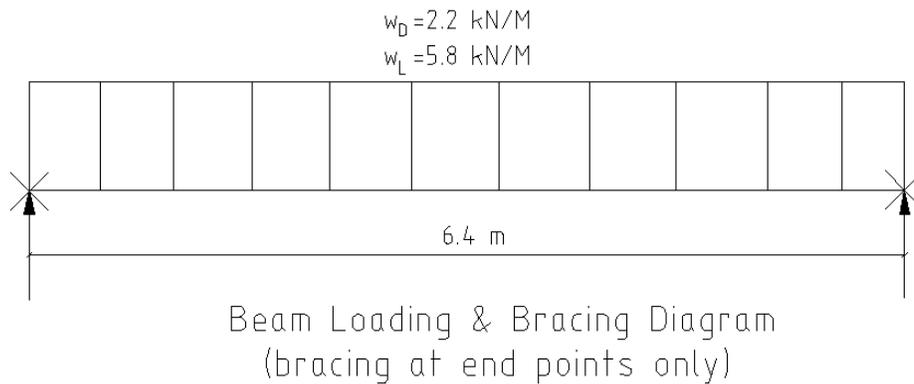
From CISC Handbook Part 4,  $M_r = 13.5 \text{ kN} - \text{m} > 8 \text{ kN} - \text{m}$       **O.K.**

$M_r$  (**S – STEEL**) = 13 kN – m (3.70% Diff.)

## EXAMPLE F.7A HSS FLEXURAL MEMBER WITH CLASS 2 FLANGES

### Given:

Select a rectangular CSA G40.21- 350W Class C HSS beam with a span of 6.4 m. The loads include a uniform dead load of 2.2 kN/m and a uniform dead load of 5.8 kN/m. Limit the live load deflection to  $L/240$ . The beam is braced at the end points only. A class 2 member was selected here to illustrate the relative ease of selecting class 2 shape from CISC *Handbook*, as compared to designing a similar shape by applying the clauses from CSA S16-14 directly, as shown in Example F.7B.



### Solution:

From CISC *Handbook* Part 6, the material properties are as follows:

CSA G40.21- 350W Class C

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From Division B Part 4 of NBCC 2015, the bending moment under factored load is:

$$w = 1.25D + 1.5L = 1.25 \times 2.2 + 1.5 \times 5.8 = 11.5 \text{ kN/m}$$

$$M_f = \frac{11.5 \text{ kN/m} \times (6.4 \text{ m})^2}{8} = 58.8 \text{ kN} \cdot \text{m}$$

### Minimum Required Moment of Inertia

The maximum live load deflection permitted is:

$$\Delta_{max} = \frac{L}{240}$$

$$= \frac{6.4m \left( \frac{1000mm}{m} \right)}{240}$$

$$= 26.67 \text{ mm}$$

Determine the minimum required  $I$  as follows.

$$\begin{aligned} I_{reqd} &= \frac{5w_L l^4}{384E\Delta_{max}} \text{ from CISC Handbook Part 5} \\ &= \frac{5 \times 5.8kN/m \times (6.4 \times 1000)^4}{384 \times (200 \times 10^3 MPa) \times 26.67mm} \\ &= 23.7 \times 10^6 \text{ mm}^4 \end{aligned}$$

### Beam Selection

Select a rectangular HSS with a minimum  $I_x$  of  $23.7 \times 10^6 \text{ mm}^4$ , using CISC Handbook Part 6, and having adequate factored resistance, using CISC Handbook Part 4.

Try an HSS254x152x4.8 oriented in the strong direction. This rectangular HSS section was purposely selected for illustration purpose because it has a class 2 flange. See CSA S16-14 Table 2 for classification criteria.

From CISC Handbook Part 6,  $I_x = 33.3 \times 10^6 \text{ mm}^4 > 23.7 \times 10^6 \text{ mm}^4$  **O.K.**

From CISC Handbook Part 4,  $M_r = 99.9kN - m > 58.8kN - m$  **O.K.**

$M_r$  (S – STEEL) = 102 kN – m (2.10% Diff.)

## EXAMPLE F.7B HSS FLEXURAL MEMBER WITH CLASS 2 FLANGES

### Given:

Notice that in Example F.7A the required information was easily determined by consulting the tables of the CISC *Handbook*. The purpose of the following calculation is to demonstrate the use of the CSA S16-14 equations to calculate the flexural resistance of an HSS member with a class 2 compression flange.

### Solution:

From CISC *Handbook* Part 6, the material properties are as follows:

CSA G40.21- 350W Class C

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

HSS254x152x4.8

$$t = 4.78 \text{ mm}$$

$$d = 254 \text{ mm}$$

$$b = 152.4 \text{ mm}$$

$$Z_x = 317 \times 10^3 \text{ mm}^3$$

### Slenderness Ratio

Based on CSA S16-14 Table 2, the maximum width-to-thickness ratio is checked as follows:

$$\frac{b_{el}}{t} = \frac{b - 4t}{t} = \frac{152.4 - 4 \times 4.78}{4.78} = 27.88 > \frac{420}{\sqrt{F_y}} = 22.45 \text{ (class 1 limit)}$$

$$< \frac{525}{\sqrt{F_y}} = 28.06 \text{ (class 2 limit)}$$

$$\frac{h}{t} = \frac{d - 4t}{t} = \frac{254 - 4 \times 4.78}{4.78} = 49.14 < \frac{1100}{\sqrt{F_y}} = 58.79 \text{ (class 1 limit)}$$

Note: According to Clause 11.3.2(b),  $b_{el}$  and  $h$  shall be the nominal outside dimension minus four times the wall thickness.

Therefore, this is a class 2 section.

*Factored Moment Resistance,  $M_r$* 

For this class 2 HSS section, when the web meets the requirement of class 1 but the flange slenderness exceeds the limit for class 1, Clause 13.5(a) shall apply as follows.

$$M_r = \phi Z F_y = 0.9 \times 317 \times 10^3 \text{ mm}^3 \times 350 \text{ MPa} = 99.86 \text{ kN} - \text{m} > 58.8 \text{ kN} - \text{m} \quad \mathbf{O.K.}$$

$$M_r (\mathbf{S} - \mathbf{STEEL}) = 102 \text{ kN} - \text{m} \text{ (2.14\% Diff.)}$$

## Chapter G Design of Members for Shear

### EXAMPLE G.1A W-SHAPE IN STRONG AXIS SHEAR

#### Given:

Determine the factored shear resistance and adequacy of a W610x92 G40.21- 350W beam using the CSA S16-14 with end shears of 214kN from dead load and 645kN from live load.

#### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 350W

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From Division B Part 4 of NBCC 2015, the shear force under factored load is:

$$V_f = 1.25D + 1.5L = 1.25 \times 214 + 1.5 \times 645 = 1235 \text{ kN}$$

#### Beam Selection

Select a W610x92 from table in CISC *Handbook* Part 5.

$$V_r = 1350 \text{ kN} - m > 1235 \text{ kN} - m \quad \mathbf{O.K.}$$

$$V_r (\mathbf{S - STEEL}) = 1369 \text{ kN} - m (1.41\% \text{ Diff.})$$

## EXAMPLE G.1B W-SHAPE IN STRONG AXIS SHEAR

### Given:

The factored shear resistance, which can be easily determined by the tabulated values of the *CISC Handbook*, can be verified by directly applying the clauses in CSA S16-14. Determine the factored shear resistance for the W-shape in Example G.1A by applying the clauses in CSA S16-14.

### Solution:

From *CISC Handbook* Part 6, the geometric properties are as follows:

W610x92

$d = 603 \text{ mm}$

$w = 10.9 \text{ mm}$

$t = 15 \text{ mm}$

Calculate  $A_w$

$$\begin{aligned} A_w &= dw \text{ from CSA S16-14 Clause 13.4.1.1} \\ &= 603\text{mm} \times 10.9\text{mm} \\ &= 6572.7 \text{ mm}^2 \end{aligned}$$

Calculate  $V_r$

$$h = d - 2t = 603 - 2 \times 15 = 573 \text{ mm}$$

$$\frac{h}{w} = \frac{573\text{mm}}{10.9\text{mm}} = 52.6 < \frac{1014}{\sqrt{350}} = 54.2$$

From Clause 13.4.1.1(a)(i), the ultimate shear stress shall be calculated as follows:

$$F_s = 0.66F_y = 0.66 \times 350 = 231 \text{ MPa}$$

$$\begin{aligned} V_r &= \phi A_w F_s \\ &= 0.9 \times 6572.7\text{mm}^2 \times 231\text{MPa} \\ &= 1366 \text{ kN} > 1235 \text{ kN} - m \quad \mathbf{O.K.} \end{aligned}$$

$$V_r (\mathbf{S - STEEL}) = 1369 \text{ kN} - m \text{ (0.22\% Diff.)}$$

**EXAMPLE G.2A C-SHAPE IN STRONG AXIS SHEAR****Given:**

Verify the factored shear resistance and adequacy of a C250×37 G40.21- 300W channel with ends shear of 78kN from dead load and 234kN from live load.

**Solution:**

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 300W

$$F_y = 300 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From Division B Part 4 of NBCC 2015, the shear force under factored load is:

$$V_f = 1.25D + 1.5L = 1.25 \times 78 + 1.5 \times 234 = 448.5 \text{ kN}$$

From CISC *Handbook* Part 5, the factored shear resistance is:

$$V_r = 607 \text{ kN} - m > 448.5 \text{ kN} - m \quad \mathbf{O.K.}$$

$$V_r (\mathbf{S - STEEL}) = 607 \text{ kN} - m \text{ (0.00\% Diff.)}$$

## EXAMPLE G.2B C-SHAPE IN STRONG AXIS SHEAR

### Given:

The factored shear resistance, which can be easily determined by the tabulated values of the CISC *Handbook*, can be verified by directly applying the clauses in CSA S16-14. Determine the factored shear resistance for the channel in Example G.2A.

### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 300W

$$F_y = 300 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

C250×37

$$d = 254 \text{ mm}$$

$$w = 13.4 \text{ mm}$$

$$T = 200 \text{ mm}$$

Calculate  $A_w$

$$\begin{aligned} A_w &= dw \text{ from CSA S16-14 Clause 13.4.1.1} \\ &= 254\text{mm} \times 13.4\text{mm} \\ &= 3403.6 \text{ mm}^2 \end{aligned}$$

Calculate  $V_r$

$$\frac{h}{w} = \frac{T}{w} = \frac{200\text{mm}}{13.4\text{mm}} = 14.9 < \frac{1014}{\sqrt{F_y}} = \frac{1014}{\sqrt{300}} = 58.5$$

From Clause 13.4.1.1(a)(i), the ultimate shear stress shall be calculated as follows:

$$F_s = 0.66F_y = 0.66 \times 300 = 198 \text{ MPa}$$

$$\begin{aligned} V_r &= \phi A_w F_s \\ &= 0.9 \times 3403.6\text{mm}^2 \times 198\text{MPa} \\ &= 606.5 \text{ kN} > 448.5 \text{ kN} - m \quad \mathbf{O.K.} \end{aligned}$$

$$V_r (\mathbf{S - STEEL}) = 607 \text{ kN} - m \text{ (0.08\% Diff.)}$$

## EXAMPLE G.4 RECTANGULAR HSS IN SHEAR

### Given:

Determine the factored shear resistance and adequacy of an HSS152x102x9.5 G40.21- 350W member with end shears of 49kN from dead load and 148kN from live load. The beam is oriented with the shear parallel to the 152mm dimension.

### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

CSA G40.21- 350W

$$F_y = 350 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

HSS152x102x9.5

$$h = 152 \text{ mm}$$

$$w = 9.53 \text{ mm}$$

$$b = 102 \text{ mm}$$

From Division B Part 4 of NBCC 2015, the shear force under factored load is:

$$V_f = 1.25D + 1.5L = 1.25 \times 49 + 1.5 \times 148 = 283 \text{ kN}$$

From the beam load table in CISC *Handbook* Part 5, the factored shear resistance is as follows:

$$V_r = 453 \text{ kN} > 283 \text{ kN} \quad \mathbf{O.K.}$$

The factored shear resistance could also be determined by applying the clauses in CSA S16-14 as follows.

### *Factored Shear Resistance*

According to Clause 11.3.2(b),  $h'$  of rectangular hollow section shall be the nominal outside dimension minus four times the wall thickness.

$$h' = h - 4w = 152\text{mm} - 4 \times 9.53\text{mm} = 113.9\text{mm}$$

Calculate  $A_w$

$$A_w = 2ht \text{ from CSA S16-14 Clause 13.4.1.1}$$

$$\begin{aligned}
 &= 2h'w \\
 &= 2 \times 113.9\text{mm} \times 9.53\text{mm} \\
 &= 2170.9 \text{ mm}^2
 \end{aligned}$$

Calculate  $V_r$

$$\frac{h}{w} = \frac{h'}{w} = \frac{113.9\text{mm}}{9.53\text{mm}} = 11.9 < \frac{1014}{\sqrt{350}} = 54.2$$

From Clause 13.4.1.1(a)(i), the ultimate shear stress shall be calculated as follows:

$$F_s = 0.66F_y = 0.66 \times 350 = 231 \text{ MPa}$$

$$\begin{aligned}
 V_r &= \phi A_w F_s \\
 &= 0.9 \times 2170.9\text{mm}^2 \times 231\text{MPa} \\
 &= 451 \text{ kN} > 283 \text{ kN} - m \quad \quad \quad \mathbf{O.K.}
 \end{aligned}$$

$$V_r (\mathbf{S - STEEL}) = 453 \text{ kN} - m \text{ (0.44\% Diff.)}$$

## Chapter H Design of Members for Combined Forces

### EXAMPLE H.2 W-SHAPE SUBJECT TO COMBINED COMPRESSION AND BENDING MOMENT ABOUT BOTH AXES

#### Given:

Using Clauses in CSA S16-14, determine if an ASTM A992 W360x147 has sufficient factored resistance to support the axial forces and moments listed as follows, obtained from a second-order analysis that includes  $P - \delta$  effects. The unbraced length is 4.3m and the member has pinned ends.  $KL_x = KL_y = L_b = 4.3m$ . This example is included primarily to illustrate the use of CSA S16-14 Clause 13.8.3.

$C_f = 1361 \text{ kN}$
$M_{fx} = 284 \text{ kN} - \text{m}$
$M_{fy} = 91 \text{ kN} - \text{m}$

Note: Assume this member is in an unbraced frame.

#### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

ASTM A992

$$F_y = 345 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From CISC *Handbook* Table 5-1, the detailed parameters of section in bending are as follows:

W360x147(Class 3)

$$b_{el}/t = 9.34$$

$$h/w = 26.0$$

From CISC *Handbook* Part 6, the geometric properties are as follows:

W360x147

$$S_x = 2570 \times 10^3 \text{ mm}^3$$

$$I_y = 167 \times 10^6 \text{ mm}^4$$

$$C_w = 4840 \times 10^9 \text{ mm}^6$$

$$J = 2230 \times 10^3 \text{ mm}^4$$

According to Clause 13.8.3, for the member in an unbraced frame, the capacity of the member shall be examined for

the overall member strength and lateral-torsional buckling strength. The member required to resist both bending moments and an axial compressive force shall be proportioned using equation  $\frac{C_f}{C_r} + \frac{U_{1x}M_{fx}}{M_{rx}} + \frac{U_{1y}M_{fy}}{M_{ry}}$ , so that the combined force and moment ratio is less than 1.  $U_{1x}$  and  $U_{1y}$  shall be taken as 1.0 for members in unbraced frames.

#### Factored Axial Compressive Resistance, $C_r$

From CISC *Handbook* Annex F Clause F.2, for a pinned-pinned condition,  $K = 1.0$ . Because the unbraced length is the same in both the x-x and y-y directions and  $r_x$  exceeds  $r_y$  for all W-shapes, y-y axis buckling will govern. From the table in CISC *Handbook* Part 4, the factored compressive resistance for a y-y axis effective length of 4300 mm is  $C_r = 4912 \text{ kN}$ , which can also be calculated according to Clause 13.3.

#### Factored Moment Resistance, $M_r$

The factored moment resistance could be calculated according to Clause 13.5(b) or obtained from the table in CISC *Handbook* Part 4. The corresponding  $M_{rx} = 798 \text{ kN} - \text{m}$  and  $M_{ry} = 281 \text{ kN} - \text{m}$ .

#### Check Strength and Stability of the Member

For the overall member strength case, the calculation shall be as follows:

$$\begin{aligned} & \frac{C_f}{C_r} + \frac{U_{1x}M_{fx}}{M_{rx}} + \frac{U_{1y}M_{fy}}{M_{ry}} \\ &= \frac{1361 \text{ kN}}{4912 \text{ kN}} + \frac{1.0 \times 284 \text{ kN} - \text{m}}{798 \text{ kN} - \text{m}} + \frac{1.0 \times 91 \text{ kN} - \text{m}}{281 \text{ kN} - \text{m}} \\ &= 0.277 + 0.356 + 0.324 \\ &= 0.957 < 1.0 \quad \text{O.K.} \end{aligned}$$

For the lateral-torsional buckling strength case, Clause 13.6(b) applies when calculating  $M_{rx}$ . Assuming loads are applied at the top flange of W shape,  $M_u$  may be determined using  $\omega_2 = 1.0$  and using an effective length, for pinned-ended beams, equal to  $1.2L$ .

$$\begin{aligned} M_u &= \frac{\omega_2 \pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} \\ &= \frac{1.0 \times \pi}{1.2 \times 4300 \text{ mm}} \times \sqrt{200 \times 10^3 \text{ MPa} \times 167 \times 10^6 \text{ mm}^4 \times 77 \times 10^3 \text{ MPa} \times 2230 \times 10^3 \text{ mm}^4} \\ &\quad \sqrt{+ \left(\frac{\pi \times (200 \times 10^3 \text{ MPa})}{1.2 \times 4300 \text{ mm}}\right)^2 \times 167 \times 10^6 \text{ mm}^4 \times 4840 \times 10^9 \text{ mm}^6} \\ &= 2563 \text{ kN} - \text{m} \end{aligned}$$

$$M_y = S_x F_y = 2570 \times 10^3 \text{ mm}^3 \times 345 \text{ MPa} = 887 \text{ kN} - \text{m}$$

Since  $M_u = 2563 \text{ kN} - \text{m} > 0.67M_y = 0.67 \times 887 \text{ kN} - \text{m} = 594 \text{ kN} - \text{m}$ , Clause 13.6(b)(i) shall apply.

$$\begin{aligned} M_{rx} &= 1.15 \phi M_y \left[ 1 - \frac{0.28 M_y}{M_u} \right] \\ &= 1.15 \times 0.9 \times 887 \text{ kN} - \text{m} \times \left[ 1 - \frac{0.28 \times 887 \text{ kN} - \text{m}}{2563 \text{ kN} - \text{m}} \right] \\ &= 829 \text{ kN} - \text{m} > \phi M_y = 0.9 \times 887 \text{ kN} - \text{m} = 798 \text{ kN} - \text{m} \end{aligned}$$

Then,  $M_{rx} = 798 \text{ kN} - \text{m}$ .

$$\begin{aligned} &\frac{C_f}{C_r} + \frac{U_{1x} M_{fx}}{M_{rx}} + \frac{U_{1y} M_{fy}}{M_{ry}} \\ &= \frac{1361 \text{ kN}}{4912 \text{ kN}} + \frac{1.0 \times 284 \text{ kN} - \text{m}}{798 \text{ kN} - \text{m}} + \frac{1.0 \times 91 \text{ kN} - \text{m}}{281 \text{ kN} - \text{m}} \\ &= 0.277 + 0.356 + 0.324 \\ &= 0.957 < 1.0 \quad \quad \quad \mathbf{O. K.} \end{aligned}$$

$$\frac{C_f}{C_r} + \frac{U_{1x} M_{fx}}{M_{rx}} + \frac{U_{1y} M_{fy}}{M_{ry}} (\mathbf{S - STEEL}) = 0.957 \text{ (0.00\% Diff.)}$$

## EXAMPLE H.4 W-SHAPE SUBJECT TO COMBINED AXIAL COMPRESSION AND FLEXURE

### Given:

Select an ASTM A992 W-shape with a 250mm nominal depth to carry axial compression forces of 17 kN from dead load and 50 kN from live load. The unbraced length is 4.3m and the ends are pinned. The member also has the following required moment resistances due to uniformly distributed loads, not including second-order effects:

$$M_{xD} = 15 \text{ kN} - m$$

$$M_{xL} = 46 \text{ kN} - m$$

$$M_{yD} = 2 \text{ kN} - m$$

$$M_{yL} = 6 \text{ kN} - m$$

The member is not subject to sidesway (no lateral translation).

### Solution:

From CISC *Handbook* Table 6-3, the material properties are as follows:

ASTM A992

$$F_y = 345 \text{ MPa}$$

$$F_u = 450 \text{ MPa}$$

From Division B Part 4 of NBCC 2015, the required resistance (not considering second-order effects) is:

$$C_f = 1.25D + 1.5L = 1.25 \times 17 + 1.5 \times 50 = 96 \text{ kN}$$

$$M_{fx} = 1.25D + 1.5L = 1.25 \times 15 + 1.5 \times 46 = 88 \text{ kN} - m$$

$$M_{fy} = 1.25D + 1.5L = 1.25 \times 2 + 1.5 \times 6 = 11.5 \text{ kN} - m$$

Try a W250x49.

From CISC *Handbook* Part 6, the geometric properties are as follows:

W250x49

$$A = 6260 \text{ mm}^2$$

$$S_x = 572 \times 10^3 \text{ mm}^3$$

$$I_x = 70.6 \times 10^6 \text{ mm}^4$$

$$r_x = 106 \text{ mm}$$

$$S_y = 150 \times 10^3 \text{ mm}^3$$

$$I_y = 15.1 \times 10^6 \text{ mm}^4$$

$$r_y = 49.2 \text{ mm}$$

$$C_w = 211 \times 10^9 \text{ mm}^6$$

$$J = 241 \times 10^3 \text{ mm}^4$$

### Check Strength and Stability of the Member

According to Clause 13.8.3, for the member in a braced frame, the capacity of the member shall be examined for cross-sectional strength, overall member strength and lateral-torsional buckling strength. The member required to resist both bending moments and an axial compressive force shall be proportioned so that the combined force and moment ratio is less than 1.

For cross-sectional strength checking, the calculation parameters shall follow the requirements in Clause 13.8.2(a).

$$C_r = \frac{\phi A F_y}{(1 + \lambda^{2n})^{\frac{1}{n}}} \text{ where } \lambda = 0$$

$$= \frac{0.9 \times 6260 \text{ mm}^2 \times 345 \text{ MPa}}{(1 + 0)^{\frac{1}{1.34}}}$$

$$= 1944 \text{ MPa}$$

$$C_{ex} = \frac{\pi^2 E I_x}{L_x^2}$$

$$= \frac{\pi^2 (200 \times 10^3 \text{ MPa}) \times 70.6 \times 10^6 \text{ mm}^4}{(4300 \text{ mm})^2}$$

$$= 7537 \text{ kN}$$

$$C_{ey} = \frac{\pi^2 E I_y}{L_y^2}$$

$$= \frac{\pi^2 (200 \times 10^3 \text{ MPa}) \times 15.1 \times 10^6 \text{ mm}^4}{(4300 \text{ mm})^2}$$

$$= 1612 \text{ kN}$$

$U_1$  accounts for the second-order effects due to the deformation of a member between its ends. According to Clause 13.8.5(b),  $\omega_1 = 1.0$ .

$$U_{1x} = \left[ \frac{\omega_1}{1 - \frac{C_f}{C_{ex}}} \right]$$

$$= \left[ \frac{1.0}{1 - \frac{96 \text{ kN}}{7537 \text{ kN}}} \right]$$

$$= 1.013 > 1.0 \quad \mathbf{O. K.}$$

$$U_{1y} = \left[ \frac{\omega_1}{1 - \frac{C_f}{C_{ey}}} \right]$$

$$= \left[ \frac{1.0}{1 - \frac{96 \text{ kN}}{1612 \text{ kN}}} \right]$$

$$= 1.063 > 1.0 \quad \mathbf{O. K.}$$

$$\frac{C_f}{C_r} + \frac{U_{1x}M_{fx}}{M_{rx}} + \frac{U_{1y}M_{fy}}{M_{ry}}$$

$$= \frac{96 \text{ kN}}{1944 \text{ kN}} + \frac{1.013 \times 88 \text{ kN} - m}{178 \text{ kN} - m} + \frac{1.063 \times 11.5 \text{ kN} - m}{46.6 \text{ kN} - m}$$

$$= 0.049 + 0.501 + 0.262$$

$$= 0.812 < 1.0 \quad \mathbf{O. K.}$$

For overall member strength checking, the calculation parameters shall follow the requirements in Clause 13.8.2(b).

$$\frac{C_f}{C_r} + \frac{U_{1x}M_{fx}}{M_{rx}} + \frac{U_{1y}M_{fy}}{M_{ry}}$$

*Factored Axial Compressive Resistance,  $C_r$*

From CISC *Handbook* Annex F Clause F.2, for a pinned-pinned condition,  $K = 1.0$ .

Because the unbraced length is the same in both the x-x and y-y directions and  $r_x$  exceeds  $r_y$  for all W-shapes, y-y axis buckling will govern.

From the table in CISC *Handbook* Part 4, the factored compressive resistance for a y-y axis effective length of 4300 mm is  $C_r = 988 \text{ kN}$ , which can also be calculated according to Clause 13.3.

*Factored Moment Resistance about the X-X Axis*

The factored moment resistance could be calculated according to Clause 13.5(b) or obtained from the table in CISC *Handbook* Part 4. The corresponding  $M_r$  are  $M_{rx} = 178 \text{ kN} - m$  and  $M_{ry} = 46.6 \text{ kN} - m$ .

$$\begin{aligned}
 &= \frac{96 \text{ kN}}{988 \text{ kN}} + \frac{1.013 \times 88 \text{ kN} - m}{178 \text{ kN} - m} + \frac{1.063 \times 11.5 \text{ kN} - m}{46.6 \text{ kN} - m} \\
 &= 0.097 + 0.501 + 0.262 \\
 &= 0.86 < 1.0 \quad \mathbf{O. K.}
 \end{aligned}$$

For the lateral-torsional buckling strength checking, the calculation parameters shall follow the requirements in Clause 13.8.2(c). Clause 13.6(b) shall apply when calculating  $M_{rx}$ . Assuming loads are applied at the top flange of W shape,  $M_u$  may be determined using  $\omega_2 = 1.0$  and using an effective length, for pinned-ended beams, equal to  $1.2L$ .

$$\begin{aligned}
 M_u &= \frac{\omega_2 \pi}{L} \sqrt{EI_y GJ + \left(\frac{\pi E}{L}\right)^2 I_y C_w} \\
 &= \frac{1.0 \times \pi}{1.2 \times 4300 \text{ mm}} \times \sqrt{200 \times 10^3 \text{ MPa} \times 15.1 \times 10^6 \text{ mm}^4 \times 77 \times 10^3 \text{ MPa} \times 241 \times 10^3 \text{ mm}^4} \\
 &\quad \sqrt{\left(\frac{\pi \times (200 \times 10^3 \text{ MPa})}{1.2 \times 4300 \text{ mm}}\right)^2 \times 15.1 \times 10^6 \text{ mm}^4 \times 211 \times 10^9 \text{ mm}^6} \\
 &= 196 \text{ kN} - m
 \end{aligned}$$

$$M_y = S_x F_y = 572 \times 10^3 \text{ mm}^3 \times 345 \text{ MPa} = 197 \text{ kN} - m$$

Since  $M_u = 196 \text{ kN} - m > 0.67M_y = 0.67 \times 197 \text{ kN} - m = 132 \text{ kN} - m$ , Clause 13.6(b)(i) shall apply.

$$\begin{aligned}
 M_{rx} &= 1.15 \phi M_y \left[1 - \frac{0.28 M_y}{M_u}\right] \\
 &= 1.15 \times 0.9 \times 197 \text{ kN} - m \times \left[1 - \frac{0.28 \times 197 \text{ kN} - m}{196 \text{ kN} - m}\right] \\
 &= 147 \text{ kN} - m < \phi M_y = 0.9 \times 197 \text{ kN} - m = 177 \text{ kN} - m
 \end{aligned}$$

Then,  $M_{rx} = 147 \text{ kN} - m$ .

$$\begin{aligned}
 &\frac{C_f}{C_r} + \frac{U_{1x} M_{fx}}{M_{rx}} + \frac{U_{1y} M_{fy}}{M_{ry}} \\
 &= \frac{96 \text{ kN}}{988 \text{ kN}} + \frac{1.013 \times 88 \text{ kN} - m}{147 \text{ kN} - m} + \frac{1.063 \times 11.5 \text{ kN} - m}{46.6 \text{ kN} - m} \\
 &= 0.097 + 0.606 + 0.262 \\
 &= 0.965 < 1.0 \quad \mathbf{O. K.}
 \end{aligned}$$

In addition, for braced frames, the member shall meet the following requirement:

$$\frac{M_{fx}}{M_{rx}} + \frac{M_{fy}}{M_{ry}} = \frac{88 \text{ kN} - m}{178 \text{ kN} - m} + \frac{11.5 \text{ kN} - m}{46.6 \text{ kN} - m} = 0.494 + 0.247 = 0.741 < 1.0 \quad \mathbf{O. K.}$$

For cross-sectional strength checking,

$$\frac{C_f}{C_r} + \frac{U_{1x}M_{fx}}{M_{rx}} + \frac{U_{1y}M_{fy}}{M_{ry}} (\mathbf{S} - \mathbf{STEEL}) = 0.813 \text{ (0.12\% Diff.)}$$

For overall member strength checking,

$$\frac{C_f}{C_r} + \frac{U_{1x}M_{fx}}{M_{rx}} + \frac{U_{1y}M_{fy}}{M_{ry}} (\mathbf{S} - \mathbf{STEEL}) = 0.861 \text{ (0.11\% Diff.)}$$

For the lateral-torsional buckling strength checking,

$$\frac{C_f}{C_r} + \frac{U_{1x}M_{fx}}{M_{rx}} + \frac{U_{1y}M_{fy}}{M_{ry}} (\mathbf{S} - \mathbf{STEEL}) = 0.967 \text{ (0.21\% Diff.)}$$

## EXAMPLE H.5A RECTANGULAR HSS TORSIONAL STRENGTH

### Given:

Determine the available torsional strength of an ASTM A500 Grade B HSS6x4x1/4.

### Solution:

From AISC *Handbook* Table 2-4, the material properties are as follows:

ASTM A500 Grade B

$$F_y = 46 \text{ ksi}$$

$$F_u = 58 \text{ ksi}$$

From AISC *Manual* Table 1-11, the geometric properties are as follows:

HSS6x4x1/4

$$h/t = 22.8$$

$$b/t = 14.2$$

$$t = 0.233 \text{ in.}$$

$$C = 10.1 \text{ in}^3$$

The available torsional strength for rectangular HSS is stipulated in AISC *Specification* Section H3.1(b).

$h/t > b/t$ , therefore,  $h/t$  governs

$$h/t \leq 2.45 \sqrt{\frac{E}{F_y}}$$

$$22.8 \leq 2.45 \sqrt{\frac{29,000 \text{ ksi}}{46 \text{ ksi}}}$$

= 61.5, therefore, use AISC *Specification* Equation H3-3

$$F_{cr} = 0.6F_y$$

$$= 0.6 \times 46 \text{ ksi}$$

$$= 27.6 \text{ ksi}$$

(Spec. Eq. H3-3)

The nominal torsional strength is,

$$T_n = F_{cr}C$$

$$= 27.6 \text{ ksi} \times 10.1 \text{ in}^3$$

$$= 279 \text{ kip} - \text{in}$$

(Spec. Eq. H3-1)

From AISC *Specification* Section H3.1, the available torsional strength is:

$$\phi_T = 0.9$$

$$\phi_T T_n = 0.9 \times 279 \text{ kip} - \text{in.} = 251 \text{ kip} - \text{in.}$$

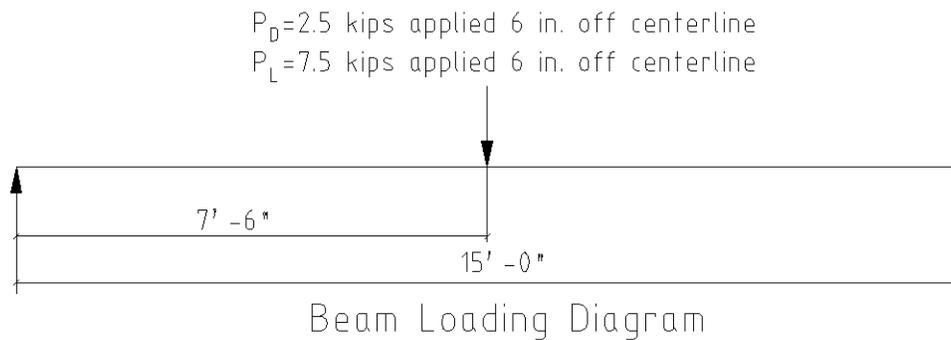
$$\phi_T T_n = (\mathbf{S} - \mathbf{STEEL}) = 250.2 \text{ kip} - \text{in} \text{ (0.32\% Diff.)}$$

Note: For more complete guidance on designing for torsion, see AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* (Seaburg and Carter, 1997).

## EXAMPLE H.6 W-SHAPE TORSIONAL STRENGTH

### Given:

This design example is taken from AISC Design Guide 9, *Torsional Analysis of Structural Steel Members*. As shown in the following diagram, an ASTM A992 W10x49 spans 15ft and supports concentrated loads at midspan that act at a 6-in. eccentricity with respect to the shear center. Determine the stresses on the cross section, the adequacy of the section to support the loads, and the maximum rotation.



The end conditions are assumed to be flexurally pinned and unrestrained for warping torsion. The eccentric load can be resolved into a torsional moment and a load applied through the shear center.

### Solution:

From AISC *Manual* Table 2-4, the material properties are as follows:

ASTM A992

$$F_y = 50 \text{ ksi}$$

$$F_u = 65 \text{ ksi}$$

From AISC *Manual* Table 1-1, the geometric properties are as follows:

W10x49

$$I_x = 272 \text{ in.}^4$$

$$S_x = 54.6 \text{ in.}^3$$

$$t_f = 0.560 \text{ in.}$$

$$t_w = 0.340 \text{ in.}$$

$$J = 1.39 \text{ in.}^4$$

$$C_w = 2,070 \text{ in.}^6$$

$$Z_x = 60.4 \text{ in.}^3$$

From the AISC Shapes Database, the additional torsional properties are as follows:

W10x49

$$S_{w1} = 33.0 \text{ in.}^4$$

$$W_{no} = 23.6 \text{ in.}^2$$

$$Q_f = 12.8 \text{ in.}^3$$

$$Q_w = 29.8 \text{ in.}^3$$

From AISC Design Guide 9 (Seaburg and Carter, 1997), the torsional property,  $a$ , is calculated as follows:

$$\begin{aligned} a &= \sqrt{\frac{EC_w}{GJ}} \\ &= \sqrt{\frac{29,000 \text{ ksi} \times 2,070 \text{ in.}^6}{11,200 \text{ ksi} \times 1.39 \text{ in.}^4}} \\ &= 62.1 \text{ in.} \end{aligned}$$

From Chapter 2 of ASCE/SEI 7, the required strength is:

LRFD
$P_u = 1.2P_D + 1.6P_L = 1.2 \times 2.50 \text{ kips} + 1.6 \times 7.50 \text{ kips} = 15.0 \text{ kips}$
$V_u = \frac{P_u}{2} = \frac{15.0 \text{ kips}}{2} = 7.50 \text{ kips}$
$M_u = \frac{P_u l}{4} = \frac{15.0 \text{ kips} \times 15.0 \text{ ft} \times 12 \text{ in./ft}}{4} = 675 \text{ kip} - \text{in.}$
$T_u = P_u e = 15.0 \text{ kips} \times 6.00 \text{ in.} = 90.0 \text{ kip} - \text{in.}$

### Normal and Shear Stresses from Flexure

The normal and shear stresses from flexure are determined from AISC Design Guide 9, as follows:

LRFD
$\sigma_{ub} = \frac{M_u}{S_x} \quad \text{(from Design Guide 9 Eq. 4.5)}$
$= \frac{675 \text{ kip} - \text{in}}{54.6 \text{ in.}^3}$
$= 12.4 \text{ ksi (compression at top, tension at bottom)}$
$\tau_{ub \text{ web}} = \frac{V_u Q_w}{I_x t_w} \quad \text{(from Design Guide 9 Eq. 4.6)}$

$$\begin{aligned}
 &= \frac{7.50 \text{ kips} \times 29.8 \text{ in.}^3}{272 \text{ in.}^4 \times 0.340 \text{ in.}} \\
 &= 2.42 \text{ ksi}
 \end{aligned}$$

$$\begin{aligned}
 \tau_{ub \text{ flange}} &= \frac{V_u Q_f}{I_x t_f} && \text{(from Design Guide 9 Eq. 4.6)} \\
 &= \frac{7.50 \text{ kips} \times 12.8 \text{ in.}^3}{272 \text{ in.}^4 \times 0.560 \text{ in.}} \\
 &= 0.630 \text{ ksi}
 \end{aligned}$$

### Torsional Stresses

The following functions are taken from AISC Design Guide 9, *Torsional Analysis of Structural Steel Members*, Appendix B, Case 3, with  $\alpha = 0.5$ .

$$\frac{l}{a} = \frac{180 \text{ in.}}{62.1 \text{ in.}} = 2.90$$

At midspan ( $z/l = 0.5$ ):

Using the graphs for  $\theta$ ,  $\theta''$ ,  $\theta'$  and  $\theta'''$ , select values

$$\text{For } \theta: \quad \theta \times \left(\frac{GJ}{T_r}\right) \left(\frac{1}{l}\right) = +0.09 \quad \text{Solve for } \theta = +0.09 \frac{T_r l}{GJ}$$

$$\text{For } \theta'': \quad \theta'' \times \left(\frac{GJ}{T_r}\right) a = -0.44 \quad \text{Solve for } \theta'' = -0.44 \frac{T_r}{GJ a}$$

$$\text{For } \theta': \quad \theta' \times \left(\frac{GJ}{T_r}\right) = 0 \quad \text{Therefore } \theta' = 0$$

$$\text{For } \theta''': \quad \theta''' \times \left(\frac{GJ}{T_r}\right) a^2 = -0.50 \quad \text{Solve for } \theta''' = -0.50 \frac{T_r}{GJ a^2}$$

At the support ( $z/l = 0$ ):

$$\text{For } \theta: \quad \theta \times \left(\frac{GJ}{T_r}\right) \left(\frac{1}{l}\right) = 0 \quad \text{Therefore } \theta = 0$$

$$\text{For } \theta'': \quad \theta'' \times \left(\frac{GJ}{T_r}\right) a = 0 \quad \text{Therefore } \theta'' = 0$$

$$\text{For } \theta': \quad \theta' \times \left(\frac{GJ}{T_r}\right) = +0.28 \quad \text{Solve for } \theta' = +0.28 \frac{T_r}{GJ}$$

$$\text{For } \theta''': \quad \theta''' \times \left(\frac{GJ}{T_r}\right) a^2 = -0.22 \quad \text{Solve for } \theta''' = -0.22 \frac{T_r}{GJ a^2}$$

In the preceding calculations, note that the applied torque is negative with the sign convention used.

Calculate  $T_r/GJ$  for use as follows:

LRFD
$\frac{T_u}{GJ} = \frac{-90.0 \text{ kip-in}}{11,200 \text{ ksi} \times 1.39 \text{ in.}^4} = -5.78 \times 10^{-3} \text{ rad/in.}$

### Shear Stresses Due to Pure Torsion

The shear stresses due to pure torsion are determined from AISC Design Guide 9 as follows:

$$\tau_t = Gt\theta' \quad \text{(Design Guide 9 Eq. 4.1)}$$

LRFD
<p>At midspan: <math>\theta' = 0, \tau_{ut} = 0</math></p> <p>At the support, for the web: <math>\tau_{ut} = 11,200 \text{ ksi} \times 0.340 \text{ in.} \times 0.28 \times \left(\frac{-5.78 \text{ rad}}{10^3 \text{ in.}}\right) = -6.16 \text{ ksi}</math></p> <p>At the support, for the flange: <math>\tau_{ut} = 11,200 \text{ ksi} \times 0.560 \text{ in.} \times 0.28 \times \left(\frac{-5.78 \text{ rad}}{10^3 \text{ in.}}\right) = -10.2 \text{ ksi}</math></p>

### Shear Stresses Due to Warping

The shear stresses due to warping are determined from AISC Design Guide 9 as follows:

$$\tau_t = \frac{-ES_{w1}\theta'''}{t_f} \quad \text{(Design Guide 9 Eq. 4.2a)}$$

LRFD
<p>At midspan:</p> $\tau_{uw} = \frac{-29,000 \text{ ksi} \times 33.0 \text{ in.}^4}{0.560 \text{ in.}} \times \left[ \frac{-0.50 \times (-5.78 \text{ rad})}{(62.1 \text{ in.})^2 \times (10^3 \text{ in.})} \right] = -1.28 \text{ ksi}$ <p>At the support:</p> $\tau_{uw} = \frac{-29,000 \text{ ksi} \times 33.0 \text{ in.}^4}{0.560 \text{ in.}} \times \left[ \frac{-0.22 \times (-5.78 \text{ rad})}{(62.1 \text{ in.})^2 \times (10^3 \text{ in.})} \right] = -0.563 \text{ ksi}$

### Normal Stresses Due to Warping

The normal stresses due to warping are determined from AISC Design Guide 9 as follows:

$$\sigma_w = EW_{no} \theta''$$

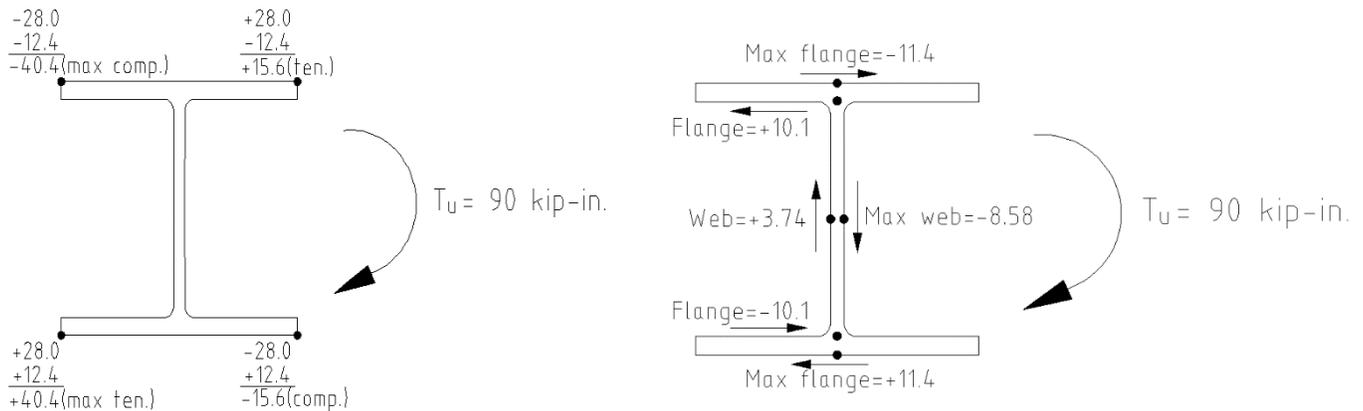
(from Design Guide 9 Eq. 4.3a)

LRFD
<p>At midspan:</p> $\sigma_{uw} = 29,000 \text{ ksi} \times 23.6 \text{ in.}^4 \times \left[ \frac{-0.44 \times (-5.78 \text{ rad})}{62.1 \text{ in.} \times (10^3 \text{ in.})} \right] = 28.0 \text{ ksi}$
<p>At the support:</p> <p>Because <math>\theta'' = 0</math>, <math>\sigma_{uw} = 0</math></p>

*Combined Stresses*

The stresses are summarized in the following table and shown in Figure H.6-1.

Summary of Stresses Due to Flexure and Torsion, ksi							
Location	LRFD						
	Normal Stresses			Shear Stresses			
	$\sigma_{uw}$	$\sigma_{ub}$	$f_{un}$	$\tau_{ut}$	$\tau_{uw}$	$\tau_{ub}$	$f_{uv}$
Midspan							
Flange	±28.0	±12.4	±40.4	0	-1.28	±0.63	-1.91
Web	---	---	---	0	---	±2.42	±2.42
Support							
Flange	0	0	0	-10.2	-0.563	±0.63	-11.4
Web	---	---	---	-6.16	---	±2.42	-8.58
Maximum			±40.4				-11.4



Normal stresses due to flexure and torsion at midspan

Shear stresses at support

**Figure H6.** Stresses due to flexure and torsion

By using LRFD method, the maximum normal stress due to flexure and torsion,  $f_{un\_max}$ , occurs at the edge of the flange at midspan and is equal to 40.4 ksi.

$$f_{un\_max}(\mathbf{S} - \mathbf{STEEL}) = 41 \text{ ksi (1.5\% Diff.)}$$

The maximum shear stress due to flexure and torsion,  $f_{uv\_max}$ , occurs in the middle of the flange at the support and is equal to 11.4 ksi.

$$f_{uv\_max}(\mathbf{S} - \mathbf{STEEL}) = 11 \text{ ksi (3.5\% Diff.)}$$

### *Available Torsional Strength*

The available torsional strength is the lowest value determined for the limit states of yielding under normal stress, shear yielding under shear stress, or buckling in accordance with AISC *Specification* Section H3.3. The nominal torsional strength due to the limit states of yielding under normal stress and shear yielding under shear stress are compared to the applicable buckling limit states.

### Buckling

For the buckling limit state, lateral-torsional buckling and local buckling must be evaluated. The nominal torsional strength due to the limit state of lateral-torsional buckling is determined as follows:

LRFD	
$C_b = 1.32$ from AISC Manual Table 3-1.	
Compute $F_n$ for a W10x49 using values from AISC <i>Manual</i> Table 3-10 with $L_b = 15.0$ ft and $C_b = 1.0$ .	
$\phi_b M_n = 204 \text{ kip-ft}$	
$F_n = F_{cr}$	(Spec. Eq. H3-9)
$= C_b \frac{\phi_b M_n}{\phi_b S_x} = 1.32 \times \frac{204 \text{ kip-ft}}{0.90 \times 54.6 \text{ in.}^3} \times \frac{12 \text{ in.}}{\text{ft}} = 65.8 \text{ ksi}$	

The limit state of local buckling does not apply because a W10x49 is compact in flexure per the user note in AISC *Specification* Section F2.

#### Yielding Under Normal Stress

The nominal torsional strength due to the limit state of yielding under normal stress is determined as follows:

$$\begin{aligned}
 F_n &= F_y && \text{(Spec. Eq. H3-7)} \\
 &= 50 \text{ ksi}
 \end{aligned}$$

Therefore, the limit state of yielding under normal stress controls over buckling. The available torsional strength for yielding under normal stress is determined as follows, from AISC *Specification* Section H3:

LRFD	
$\phi_T = 0.90$	
$\phi_T F_n = 0.90 \times 50 \text{ ksi} = 45.0 \text{ ksi} > 40.4 \text{ ksi}$	<b>O. K.</b>

#### Shear Yielding Under Shear Stress

The nominal torsional strength due to the limit state of shear yielding under shear stress is:

$$\begin{aligned}
 F_n &= 0.6F_y && \text{(Spec. Eq. H3-8)} \\
 &= 0.6 \times 50 \text{ ksi} \\
 &= 30 \text{ ksi}
 \end{aligned}$$

The limit state of shear yielding under shear stress controls over buckling. The available torsional strength for shear yielding under shear stress determined as follows, from AISC *Specification* Section H3:

LRFD	
$\phi_T = 0.90$	
$\phi_T F_n = 0.90 \times 0.6 \times 50 \text{ ksi} = 27.0 \text{ ksi} > 11.4 \text{ ksi}$	<b>O. K.</b>

#### *Maximum Rotation at Service Load*

The maximum rotation occurs at midspan. The service load torque is:

$$\begin{aligned}
 T &= Pe \\
 &= -(2.50 \text{ kips} + 7.50 \text{ kips}) \times 6.00 \text{ in.} \\
 &= -60.0 \text{ kip} - \text{in}
 \end{aligned}$$

From AISC Design Guide 9, Appendix B, Case 3 with  $\alpha = 0.5$ , the maximum rotation is:

$$\begin{aligned}
 \theta &= +0.09 \frac{Tl}{GJ} \\
 &= \frac{0.09 \times (-60.0 \text{ kip} - \text{in}) \times 180 \text{ in.}}{11,200 \text{ ksi} \times 1.39 \text{ in.}^4} \\
 &= -0.0624 \text{ rads or } -3.58^\circ
 \end{aligned}$$

$$\theta(S - \text{STEEL}) = -0.0661 (5.9\% \text{ Diff.})$$

See AISC Design Guide 9, *Torsional Analysis of Structural Steel Members* for additional guidance.